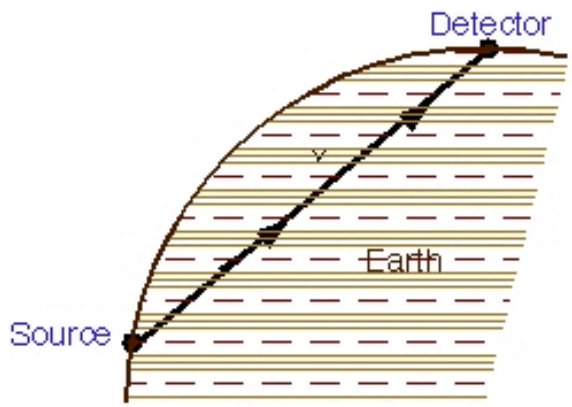
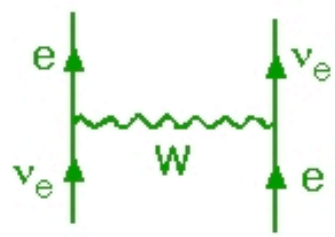
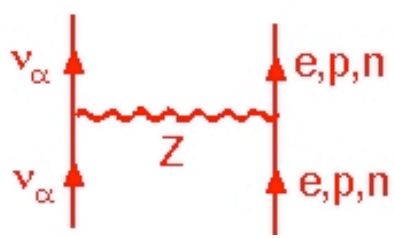
Neutrino Flavor Change in Matter



Coherent forward scattering from ambient matter can have a big effect.

Interaction





Interaction Potential Energy

$$V_W = +\sqrt{2} G_F N_e \qquad (- \text{ for } \overline{\nu_e})$$

#e/vol

$$V_Z = -\frac{\sqrt{2}}{2} G_F N_n \qquad (+ \text{ for } \overline{\nu_\alpha})$$

#n/vol

Neutrino propagation in matter is conveniently treated via a Schrödinger Equation:

$$i\frac{\partial}{\partial t}\nu(t) = H\,\nu(t)$$

Matrix in flavor space

Multi-component in flavor space

To illustrate, we describe the case —

When Only Two Neutrinos Count

Amp. to be a ν_e

$$\nu(t) = \begin{bmatrix} f_e(t) \\ f_\mu(t) \end{bmatrix} \quad ; \quad H = \begin{array}{c} \\ \nu_e \\ \nu_\mu \end{array} \begin{array}{c} \begin{array}{cc} \nu_e & \nu_\mu \end{array} \\ \begin{bmatrix} \sim & \sim \\ \sim & \sim \end{bmatrix} \end{array}$$

Amp. to be a ν_μ

A 2x2 matrix in ν_e-ν_μ space

$$i\frac{\partial}{\partial t}\begin{bmatrix} f_e(t) \\ f_\mu(t) \end{bmatrix} = \begin{bmatrix} & & \\ & H & \\ & & \end{bmatrix}\begin{bmatrix} f_e(t) \\ f_\mu(t) \end{bmatrix}$$

In general

In vacuum

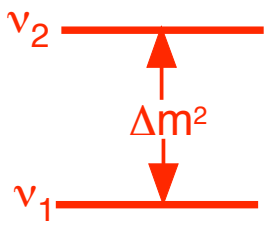
$$< \nu_\alpha|H|\nu_\beta > = < \sum_i U^*_{\alpha i}\nu_i|H|\sum_j U^*_{\beta j}\nu_j > = \sum_j U_{\alpha j}U^*_{\beta j}\sqrt{p^2 + m_j^2}$$

Momentum of the beam

In flavor change, only relative phases, hence relative energies, matter.

$\therefore$ In H, any multiple of the Identity Matrix I may be omitted.

In Vacuum



$$U = \begin{array}{c c} & \begin{array}{cc} \nu_1 & \nu_2 \end{array} \\ \begin{array}{c} \nu_e \\ \nu_\mu \end{array} & \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \end{array} \quad ; \quad \begin{array}{l} \nu_e = \nu_1 \cos\theta + \nu_2 \sin\theta \\ \nu_\mu = \nu_1(-\sin\theta) + \nu_2 \cos\theta \end{array}$$

It follows that, omitting a piece ∝ I,

$$H_{\mathrm{Vac}} = \frac{\Delta m^2}{4E} \begin{bmatrix} -\cos 2\theta & \sin 2\theta \\ \sin 2\theta & \cos 2\theta \end{bmatrix} .$$

With Schrödinger's Equation, this gives the usual $P(\nu_e \to \nu_\mu)$.

The eigenvalues of H_{Vac} are —

$$\pm\frac{\Delta m^2}{4E} \equiv \pm\lambda \quad .$$

With $c \equiv \cos\theta$, $s \equiv \sin\theta$,

$$\nu_e = \nu_1 c + \nu_2 s \xrightarrow{t} \nu(t) = \nu_1 c e^{i\lambda t} + \nu_2 s e^{-i\lambda t}$$

$$P(\nu_e \to \nu_\mu) = \quad |<\nu_\mu|\nu(t)>|^2 = |sc(-e^{i\lambda t} + e^{-i\lambda t})|^2$$

$$= \sin^2 2\theta \sin^2(\Delta m^2 \frac{L}{4E})$$

In Matter

$$H_M = H_{\rm Vac} + \quad V_W \begin{array}{c} \begin{array}{cc} \nu_e & \nu_\mu \end{array} \\ \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \end{array} \begin{array}{c} \nu_e \\ \nu_\mu \end{array} + V_Z \underbrace{\begin{array}{c} \begin{array}{cc} \nu_e & \nu_\mu \end{array} \\ \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \end{array} \begin{array}{c} \nu_e \\ \nu_\mu \end{array}}_{\propto \text{ I, so drop}}$$

$$H_M = H_{\rm Vac} + \frac{V_W}{2} \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} + \frac{V_W}{2} \overbrace{\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}}$$

$$H_M = \frac{\Delta m^2}{4E} \begin{bmatrix} -(\cos 2\theta - x) & \sin 2\theta \\ \sin 2\theta & \cos 2\theta - x \end{bmatrix} ,$$

with $\quad x \equiv \dfrac{V_W/2}{\Delta m^2/4E} = \dfrac{2\sqrt{2} G_F N_e E}{\Delta m^2} \quad .$

The Effective Splitting and Mixing in Matter

If we define —

$$\Delta m_M^2 \equiv \Delta m^2 \sqrt{\sin^2 2\theta + (\cos 2\theta - x)^2}$$

and

$$\sin^2 2\theta_M \equiv \frac{\sin^2 2\theta}{\sin^2 2\theta + (\cos 2\theta - x)^2} \quad ,$$

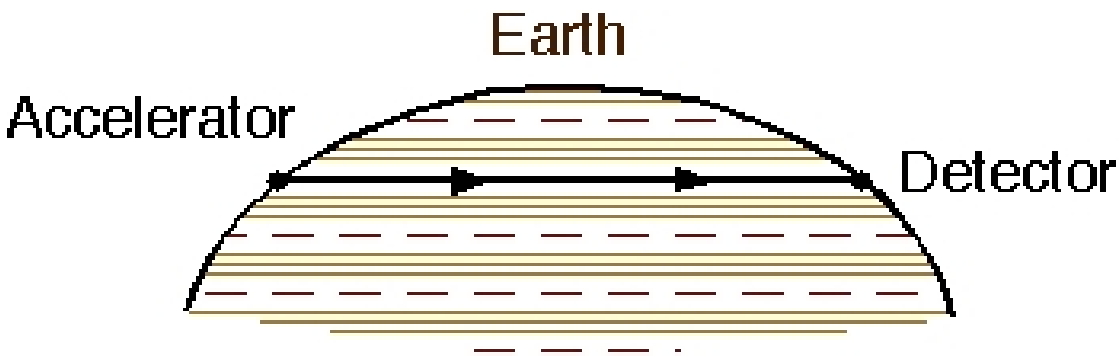
then

$$H_M = \frac{\Delta m_M^2}{4E} \begin{bmatrix} -\cos 2\theta_M & \sin 2\theta_M \\ \sin 2\theta_M & \cos 2\theta_M \end{bmatrix} \quad .$$

This is H_{Vac} with $(\Delta m^2, \theta) \rightarrow (\Delta m_M{}^2, \theta_M)$.

Thus, $\Delta m_M{}^2$ and θ_M are the effective splitting and mixing angle in matter.

Travel Through the Earth



The matter density encountered en route is ~ constant.

Thus, H_M is position-independent, just like H_{Vac}.

Therefore, in the earth (but not too deep),

$$P_M(\nu_e \to \nu_\mu) = \sin^2 2\theta_M \sin^2(\Delta m_M^2 \frac{L}{4E})$$

In matter

The Size and Consequence of the Matter Effect

The matter effect depends on —

$$x = \frac{2\sqrt{2}G_F N_e E}{\Delta m^2} \propto E \quad .$$

The denominator contains a Sign

In the earth's mantle, for $|\Delta m^2| = |\Delta m^2(\text{atmospheric})|$ $\cong 2.7 \times 10^{-3}\ \text{eV}^2$,

$$|x| \simeq \frac{E}{12\text{GeV}} \quad .$$

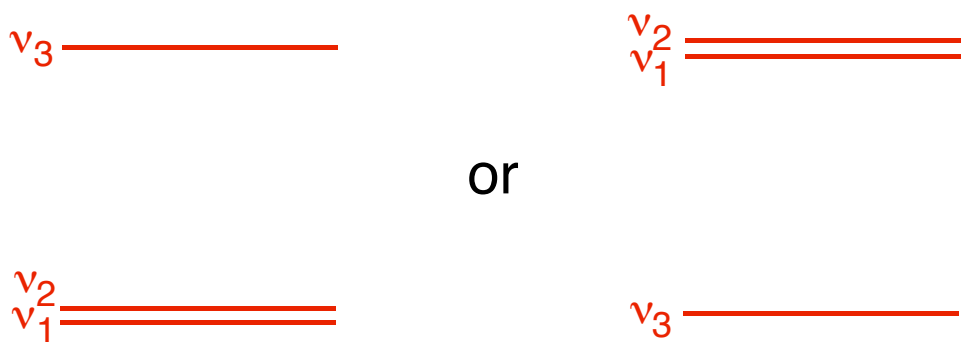
Since $V_W(\bar{\nu}) = -V_W(\nu)$, $x(\bar{\nu}) = -x(\nu)$.

Thus $\overline{\Delta m_M}^2 \neq \Delta m_M{}^2$ and $\sin^2 2\bar{\theta}_M \neq \sin^2 2\theta_M$.

The matter effect causes an asymmetry between $\bar{\nu}$ and ν oscillation. This must be separated from the genuine $\not{CP}$ asymmetry.

When Three Neutrinos Count

Neglecting the possibility of *more* than 3 neutrinos, the (Mass)^2 spectrum looks like—



with $|\Delta m^2_{21}/\Delta m^2_{31}| \sim 1/35$.

The Mixing Matrix

Atmospheric Cross-Mixing Solar

$$U = \begin{bmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{bmatrix} \times \begin{bmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{bmatrix} \times \begin{bmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\times \begin{bmatrix} e^{i\alpha_1/2} & 0 & 0 \\ 0 & e^{i\alpha_2/2} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$c_{ij} \equiv \cos\theta_{ij}$
$s_{ij} \equiv \sin\theta_{ij}$

Majorana ~~CP~~ phases

$\theta_{12} \approx \theta_{sol} \approx 34°$, $\theta_{23} \approx \theta_{atm} \approx 37\text{-}53°$, $\theta_{13} \lesssim 10°$

δ would lead to $P(\overline{\nu}_\alpha \to \overline{\nu}_\beta) \neq P(\nu_\alpha \to \nu_\beta)$. ~~CP~~

But note the crucial role of $s_{13} \equiv \sin\theta_{13}$.

The Majorana $\not{CP}$ Phases

The phase α_i is associated with neutrino mass eigenstate ν_i:

$U_{\alpha i} = U^0_{\alpha i} \exp(i\alpha_i/2)$ for all flavors α.

$$\mathrm{Amp}(\nu_\alpha \rightarrow \nu_\beta) = \sum_i U_{\alpha i}^* \exp(-\, i m_i^2 L/2E)\, U_{\beta i}$$

is insensitive to the Majorana phases α_i.

Only the phase δ can cause CP violation in neutrino oscillation.

There Is Nothing Special About θ_{13}

All mixing angles must be nonzero for ~~CP~~.

For example —

$$P(\bar{\nu}_\mu \to \bar{\nu}_e) - P(\nu_\mu \to \nu_e) = 2\cos\theta_{13}\sin 2\theta_{13}\sin 2\theta_{12}\sin 2\theta_{23}\sin\delta$$
$$\times \sin\left(\Delta m^2{}_{31}\frac{L}{4E}\right)\sin\left(\Delta m^2{}_{32}\frac{L}{4E}\right)\sin\left(\Delta m^2{}_{21}\frac{L}{4E}\right)$$

In the factored form of U, one can put δ next to θ_{12} instead of θ_{13}.

Accelerator $\overset{(-)}{\nu}$ Oscillation Probabilities

With $\alpha \equiv \Delta m_{21}^2 / \Delta m_{31}^2$, $\Delta \equiv \dfrac{\Delta m_{31}^2 L}{4E}$, and $x \equiv \dfrac{2\sqrt{2} G_F N_e E}{\Delta m_{31}^2}$ —

$$P\left[\nu_\mu \to \nu_e\right] \cong \sin^2 2\theta_{13}\, T_1 - \alpha \sin 2\theta_{13}\, T_2 + \alpha \sin 2\theta_{13}\, T_3 + \alpha^2 T_4 \;;$$

$$T_1 = \sin^2\theta_{23} \frac{\sin^2\left[(1-x)\Delta\right]}{(1-x)^2}, \quad T_2 = \sin\delta \sin 2\theta_{12} \sin 2\theta_{23} \sin\Delta \frac{\sin(x\Delta)}{x} \frac{\sin\left[(1-x)\Delta\right]}{(1-x)},$$

$$T_3 = \cos\delta \sin 2\theta_{12} \sin 2\theta_{23} \cos\Delta \frac{\sin(x\Delta)}{x} \frac{\sin\left[(1-x)\Delta\right]}{(1-x)}, \quad T_4 = \cos^2\theta_{23} \sin^2 2\theta_{12} \frac{\sin^2(x\Delta)}{x^2}$$

$$P\left[\bar{\nu}_\mu \to \bar{\nu}_e\right] = P\left[\nu_\mu \to \nu_e\right] \text{ with } \delta \to -\delta \text{ and } x \to -x.$$

(Cervera *et al.*, Freund, Akhmedov *et al.*)

Separating $\not{CP}$ From the Matter Effect

Genuine $\not{CP}$ and the matter effect both lead to a difference between ν and $\bar{\nu}$ oscillation.

But genuine $\not{CP}$ and the matter effect depend quite differently from each other on L and E.

To disentangle them, one must make oscillation measurements at different L and/or E.

Strategies

The matter-effect parameter x has $|x| \approx E/12$ GeV.

At L/E of the 1st "atmospheric" oscillation peak, and $E \sim 1$ GeV, the effect of matter on the ***neutrino*** atmospheric oscillation term ($\sin^2 2\theta_{13}\, T_1$) is —

$$1/(1-x)^2 \cong 1 \pm (E/6\,\mathrm{GeV})$$

Normal

Inverted

At fixed L/E, genuine ~~CP~~ effects do not change with E, but the matter effect grows, enhancing (suppressing) the oscillation if the hierarchy is Normal (Inverted).

If $E \rightarrow E/3$ at fixed L, we go from the 1st atmospheric oscillation peak to the 2nd one.

When $E \rightarrow E/3$ at fixed L, *the matter effect is reduced by a factor of 3*, *but* $\not{CP}$ *is tripled*.

The MSW Effect

Since —

$$\sin^2 2\theta_M = \frac{\sin^2 2\theta}{\sin^2 2\theta + (\cos 2\theta - x)^2} \quad ,$$

even a tiny vacuum mixing $\sin^2 2\theta$ can be amplified into a near-maximal in-matter mixing $\sin^2 2\theta_M$ if

$$x \cong \cos 2\theta \quad .$$

This is the "resonant" version of the —

Mikheyev Smirnov Wolfenstein Effect.

This is *NOT* what happens in the sun!