

Electroweak interactions of quarks (one generation)

- Left-handed doublet

$$\begin{array}{rcccl} & I_3 & Q & Y = 2(Q - I_3) & \\ L_q = \begin{pmatrix} u \\ d \end{pmatrix}_L & \frac{1}{2} & +\frac{2}{3} & \frac{1}{3} & \\ & -\frac{1}{2} & -\frac{1}{3} & & \end{array}$$

- two right-handed singlets

$$\begin{array}{rcccl} & I_3 & Q & Y = 2(Q - I_3) & \\ R_u = u_R & 0 & +\frac{2}{3} & +\frac{4}{3} & \\ R_d = d_R & 0 & -\frac{1}{3} & -\frac{2}{3} & \end{array}$$

Electroweak interactions of quarks

- CC interaction

$$\mathcal{L}_{W-q} = \frac{-g}{2\sqrt{2}} \left[\bar{u}_e \gamma^\mu (1 - \gamma_5) d W_\mu^+ + \bar{d} \gamma^\mu (1 - \gamma_5) u W_\mu^- \right]$$

identical in form to $\mathcal{L}_{W-\ell}$: universality \Leftrightarrow weak isospin

- NC interaction

$$\mathcal{L}_{Z-q} = \frac{-g}{4 \cos \theta_W} \sum_{i=u,d} \bar{q}_i \gamma^\mu [L_i(1 - \gamma_5) + R_i(1 + \gamma_5)] q_i Z_\mu$$

$$L_i = \tau_3 - 2Q_i \sin^2 \theta_W \quad R_i = -2Q_i \sin^2 \theta_W$$

equivalent in form (not numbers) to $\mathcal{L}_{Z-\ell}$

Trouble in Paradise

Universal $u \leftrightarrow d$, $\nu_e \leftrightarrow e$ not quite right

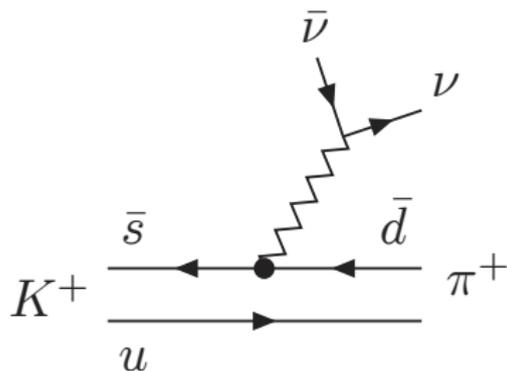
$$\text{Good: } \begin{pmatrix} u \\ d \end{pmatrix}_L \rightarrow \text{Better: } \begin{pmatrix} u \\ d_\theta \end{pmatrix}_L$$

$$d_\theta \equiv d \cos \theta_C + s \sin \theta_C \quad \cos \theta_C = 0.9736 \pm 0.0010$$

“Cabibbo-rotated” doublet perfects CC interaction (up to small third-generation effects) but \Rightarrow serious trouble for NC

$$\begin{aligned} \mathcal{L}_{Z-q} = & \frac{-g}{4 \cos \theta_W} Z_\mu \{ \bar{u} \gamma^\mu [L_u(1 - \gamma_5) + R_u(1 + \gamma_5)] u \\ & + \bar{d} \gamma^\mu [L_d(1 - \gamma_5) + R_d(1 + \gamma_5)] d \cos^2 \theta_C \\ & + \bar{s} \gamma^\mu [L_d(1 - \gamma_5) + R_d(1 + \gamma_5)] s \sin^2 \theta_C \\ & + \bar{d} \gamma^\mu [L_d(1 - \gamma_5) + R_d(1 + \gamma_5)] s \sin \theta_C \cos \theta_C \\ & + \bar{s} \gamma^\mu [L_d(1 - \gamma_5) + R_d(1 + \gamma_5)] d \sin \theta_C \cos \theta_C \} \end{aligned}$$

Strangeness-changing NC interactions highly suppressed!



BNL E-787/E-949 has three
 $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ candidates, with
 $\mathcal{B}(K^+ \rightarrow \pi^+ \nu \bar{\nu}) = 1.5_{-0.9}^{+1.3} \times 10^{-10}$
Phys. Rev. Lett. **93**, 031801 (2004)

(SM: 0.78 ± 0.11 : U. Haisch, hep-ph/0605170)

Glashow–Iliopoulos–Maiani

two LH doublets: $\begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L$ $\begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix}_L$ $\begin{pmatrix} u \\ d_\theta \end{pmatrix}_L$ $\begin{pmatrix} c \\ s_\theta \end{pmatrix}_L$

$$(s_\theta = s \cos \theta_C - d \sin \theta_C)$$

+ right-handed singlets, $e_R, \mu_R, u_R, d_R, c_R, s_R$

Required new charmed quark, c

Cross terms vanish in \mathcal{L}_{Z-q} ,

$$\frac{-ig}{4 \cos \theta_W} \gamma_\lambda [(1 - \gamma_5)L_i + (1 + \gamma_5)R_i] ,$$

$$L_i = \tau_3 - 2Q_i \sin^2 \theta_W \quad R_i = -2Q_i \sin^2 \theta_W$$

flavor-diagonal interaction!

Straightforward generalization to n quark doublets

$$\mathcal{L}_{W-q} = \frac{-g}{2\sqrt{2}} [\bar{\Psi} \gamma^\mu (1 - \gamma_5) \mathcal{O} \Psi W_\mu^+ + \text{h.c.}]$$

composite $\Psi = \begin{pmatrix} u \\ c \\ \vdots \\ d \\ s \\ \vdots \end{pmatrix}$ flavor structure $\mathcal{O} = \begin{pmatrix} 0 & U \\ 0 & 0 \end{pmatrix}$

U : unitary quark mixing matrix

Weak-isospin part: $\mathcal{L}_{Z-q}^{\text{iso}} = \frac{-g}{4 \cos \theta_W} \bar{\Psi} \gamma^\mu (1 - \gamma_5) [\mathcal{O}, \mathcal{O}^\dagger] \Psi$

Since $[\mathcal{O}, \mathcal{O}^\dagger] = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix} \propto \tau_3$

\Rightarrow NC interaction is flavor-diagonal

General $n \times n$ mixing matrix U : $n(n-1)/2$ real \angle , $(n-1)(n-2)/2$ complex phases

3×3 (Cabibbo–Kobayashi–Maskawa): 3 \angle + 1 phase

\Rightarrow \mathcal{CP} violation

Successful predictions of $SU(2)_L \otimes U(1)_Y$ theory:

- neutral-current interactions
- necessity of charm
- existence and properties of W^\pm and Z^0

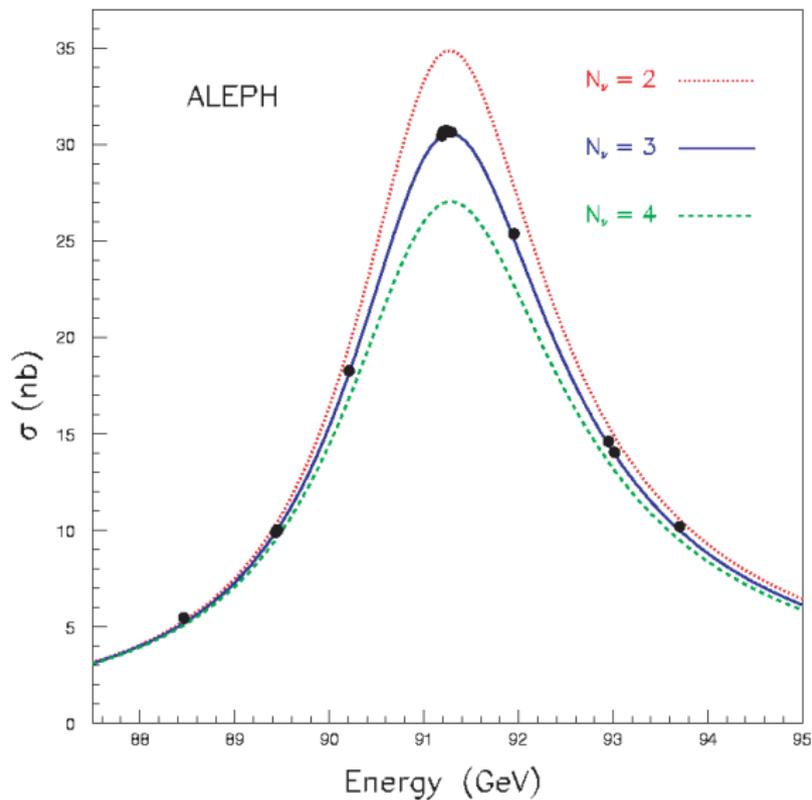
+ a decade of precision EW tests (one-per-mille)

M_W	$80\,398 \pm 25$ MeV
Γ_W	$2\,140 \pm 60$ MeV
M_Z	$91\,187.6 \pm 2.1$ MeV
Γ_Z	$2\,495.2 \pm 2.3$ MeV
$\sigma_{\text{hadronic}}^0$	41.541 ± 0.037 nb
Γ_{hadronic}	$1\,744.4 \pm 2.0$ MeV
Γ_{leptonic}	83.984 ± 0.086 MeV
$\Gamma_{\text{invisible}}$	499.0 ± 1.5 MeV

$$\Gamma_{\text{invisible}} \equiv \Gamma_Z - \Gamma_{\text{hadronic}} - 3\Gamma_{\text{leptonic}}$$

$$\text{light } \nu : N_\nu = \Gamma_{\text{invisible}}/\Gamma^{\text{SM}}(Z \rightarrow \nu_i \bar{\nu}_i) = 2.994 \pm 0.012 \quad (\nu_e, \nu_\mu, \nu_\tau)$$

Three light neutrinos



$\nu_\mu N \rightarrow \mu^- + \text{anything}$: *influence of W propagator*

$$\frac{d^2\sigma}{dx dy} = \frac{2G_F^2 M E_\nu}{\pi} \left(\frac{M_W^2}{Q^2 + M_W^2} \right)^2 [xq(x, Q^2) + x\bar{q}(x, Q^2)(1-y)^2]$$

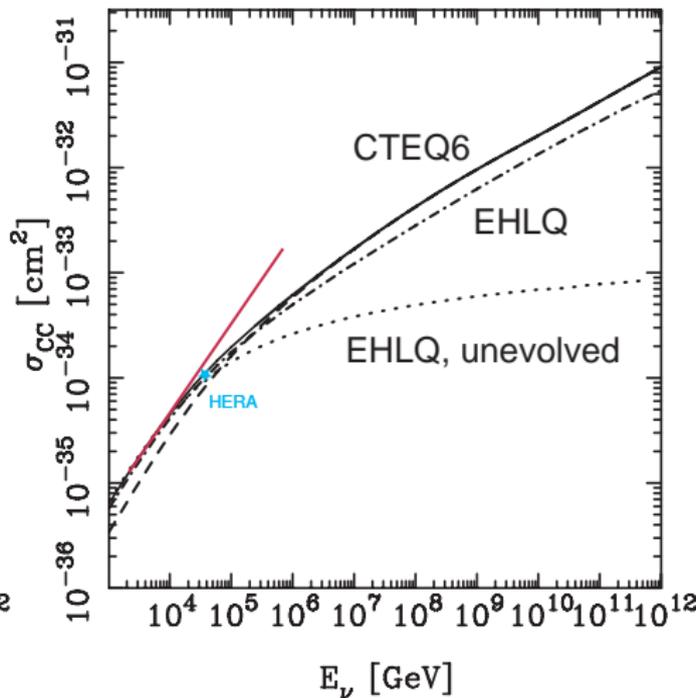
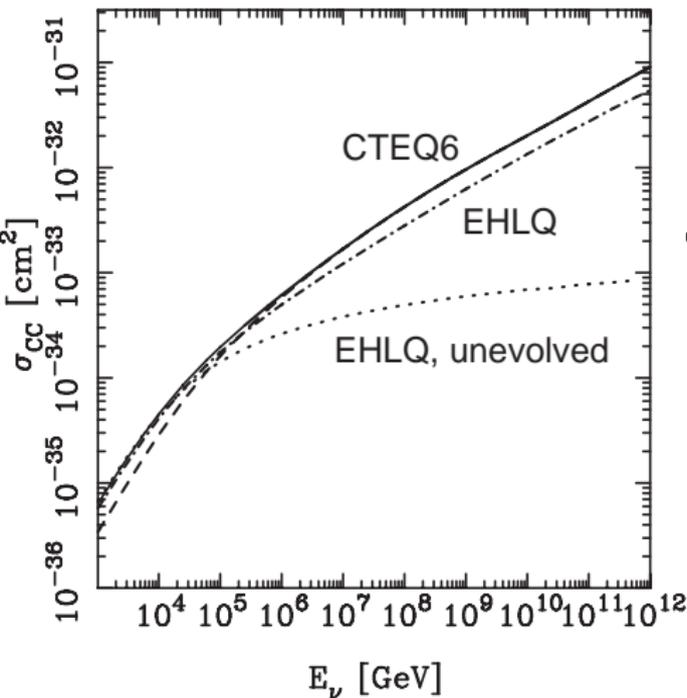
$$q(x, Q^2) = \frac{u_v(x, Q^2) + d_v(x, Q^2)}{2} + \frac{u_s(x, Q^2) + d_s(x, Q^2)}{2} + s_s(x, Q^2) + b_s(x, Q^2)$$

$$\bar{q}(x, Q^2) = \frac{u_s(x, Q^2) + d_s(x, Q^2)}{2} + c_s(x, Q^2) + t_s(x, Q^2),$$

... isoscalar nucleon

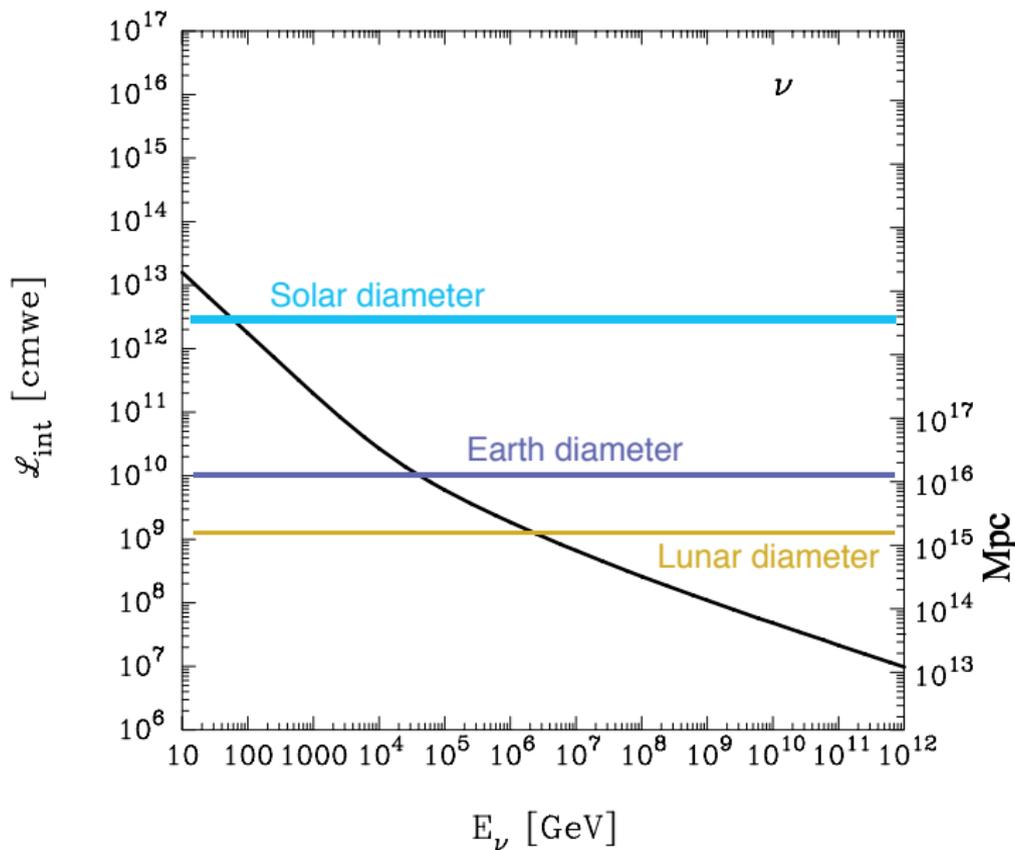
$$x = Q^2/2M\nu \quad y \equiv \nu/E_\nu \quad \nu \equiv E_\nu - E_\mu$$

$\nu N \rightarrow \mu + \dots$ Cross Sections

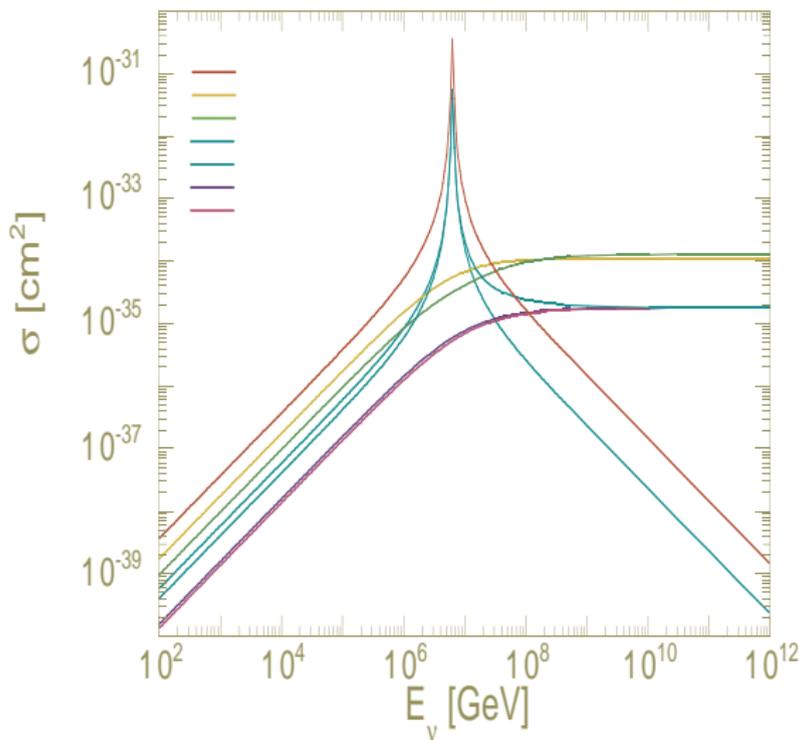


M. H. Reno

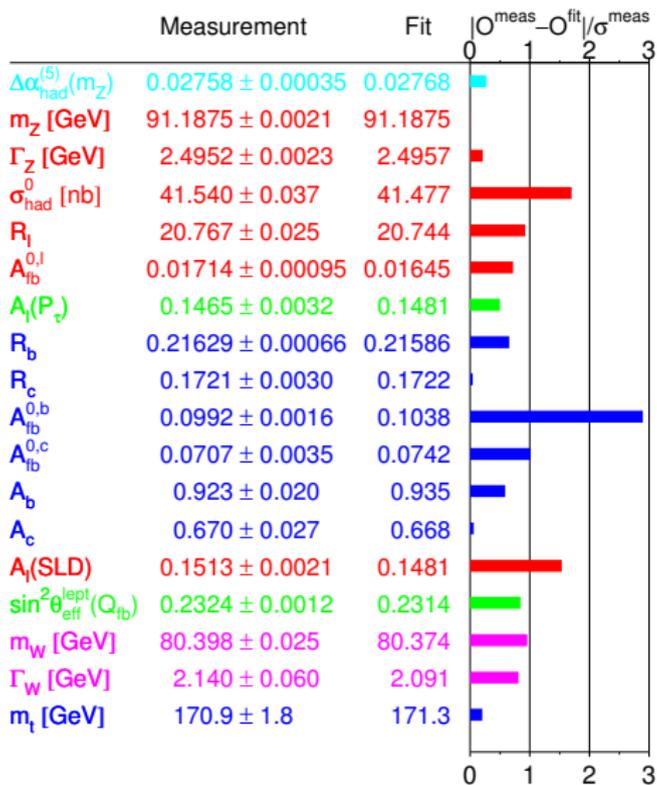
$\nu N \rightarrow \mu + \dots$ Interaction Lengths



νe cross sections ...



At low energies: $\sigma(\bar{\nu}_e e \rightarrow \text{hadrons}) > \sigma(\nu_\mu e \rightarrow \mu \nu_e) > \sigma(\nu_e e \rightarrow \nu_e e) > \sigma(\bar{\nu}_e e \rightarrow \bar{\nu}_\mu \mu) > \sigma(\bar{\nu}_e e \rightarrow \bar{\nu}_e e) > \sigma(\nu_\mu e \rightarrow \nu_\mu e) > \sigma(\bar{\nu}_\mu e \rightarrow \bar{\nu}_\mu e)$

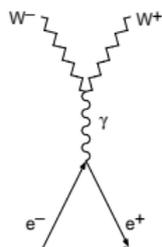


LEP Electroweak Working Group, Winter 2007

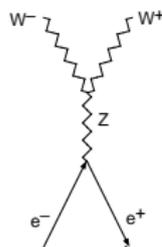
Why a Higgs boson must exist

▷ Role in canceling high-energy divergences

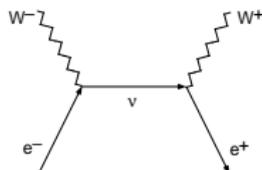
S -matrix analysis of $e^+e^- \rightarrow W^+W^-$



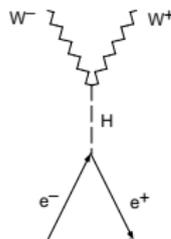
(a)



(b)



(c)

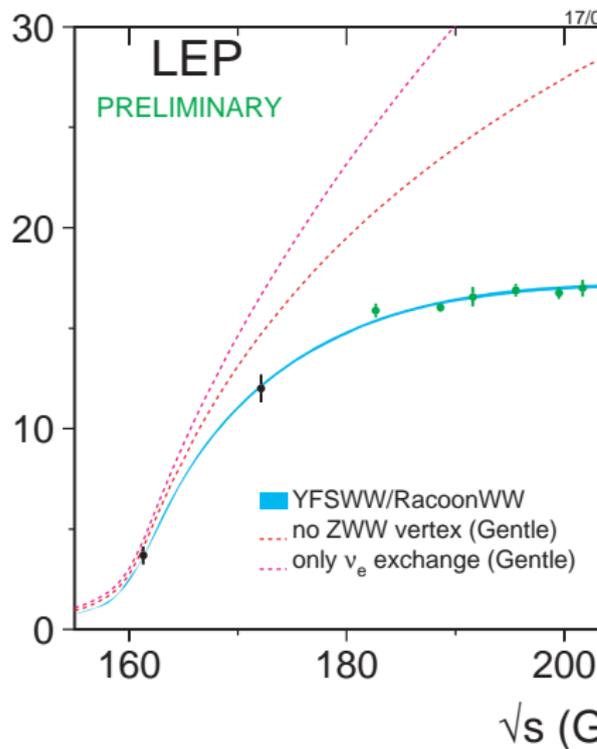
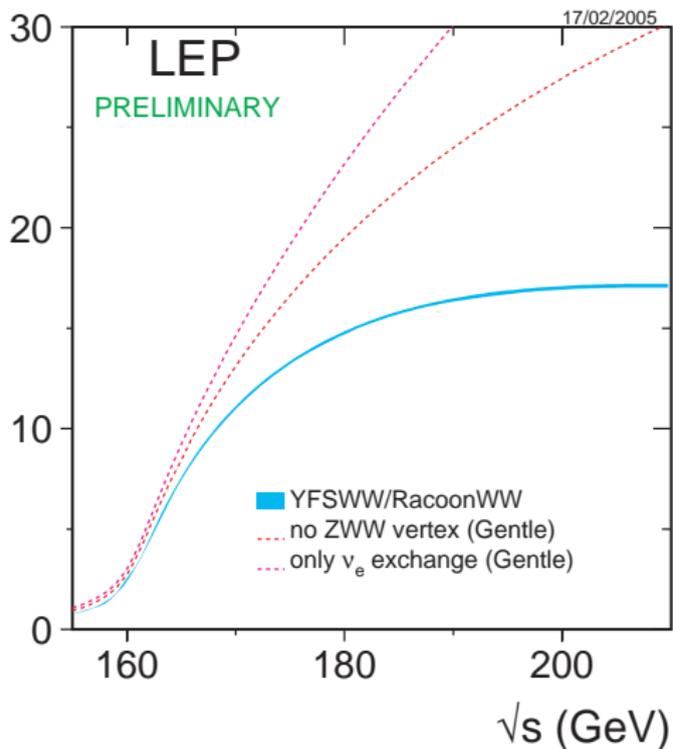


(d)

Individual $J = 1$ partial-wave amplitudes $\mathcal{M}_\gamma^{(1)}$, $\mathcal{M}_Z^{(1)}$, $\mathcal{M}_\nu^{(1)}$ have unacceptable high-energy behavior ($\propto s$)

... But sum is well-behaved

“Gauge cancellation” observed at LEP2 (Tevatron)



$J = 0$ amplitude exists because electrons have mass, and can be found in “wrong” helicity state

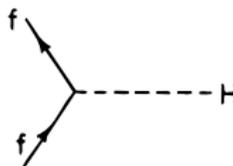
$$\mathcal{M}_\nu^{(0)} \propto s^{\frac{1}{2}} : \text{unacceptable HE behavior}$$

(no contributions from γ and Z)

This divergence is canceled by the Higgs-boson contribution

$$\Rightarrow He\bar{e} \text{ coupling must be } \propto m_e,$$

because “wrong-helicity” amplitudes $\propto m_e$



A Feynman diagram showing two incoming fermion lines (labeled 'f') merging into a Higgs boson (H) line. The fermion lines are solid with arrows pointing towards the vertex, and the Higgs line is dashed and points to the right.

$$\frac{-im_f}{v} = -im_f(G_F\sqrt{2})^{1/2}$$

If the Higgs boson did not exist, something else would have to cure divergent behavior

If gauge symmetry were unbroken . . .

- no Higgs boson
- no longitudinal gauge bosons
- no extreme divergences
- no wrong-helicity amplitudes

. . . and no viable low-energy phenomenology

In spontaneously broken theory . . .

- gauge structure of couplings eliminates the most severe divergences
- lesser—but potentially fatal—divergence arises because the electron has mass . . . due to the Higgs mechanism
- SSB provides its own cure—the Higgs boson

Similar interplay & compensation *must exist* in any acceptable theory

EWSB: another path?

Modeled EWSB on Ginzburg–Landau description of superconducting phase transition;

... had to introduce new, elementary scalars

GL is not the last word on superconductivity:

dynamical Bardeen–Cooper–Schrieffer theory

The elementary fermions – **electrons** – and gauge interactions – **QED** – needed to generate the scalar bound states are already present in the case of superconductivity.

Could a scheme of similar economy account for EWSB?

$SU(3)_c \otimes SU(2)_L \otimes U(1)_Y + \text{massless } u \text{ and } d$

(treat $SU(2)_L \otimes U(1)_Y$ as perturbation)

$m_u = m_d = 0$:

QCD has exact $SU(2)_L \otimes SU(2)_R$ chiral symmetry.

At an energy scale $\sim \Lambda_{\text{QCD}}$, strong interactions become strong, fermion condensates appear, and

$$SU(2)_L \otimes SU(2)_R \rightarrow SU(2)_V$$

\rightsquigarrow 3 Goldstone bosons, one for each broken generator:
3 massless pions (Nambu)

Broken generators: 3 axial currents; couplings to π measured by pion decay constant f_π .

Turn on $SU(2)_L \otimes U(1)_Y$: EW gauge bosons couple to axial currents, acquire masses of order $\sim gf_\pi$.

$$\mathcal{M}^2 = \begin{pmatrix} g^2 & 0 & 0 & 0 \\ 0 & g^2 & 0 & 0 \\ 0 & 0 & g^2 & gg' \\ 0 & 0 & gg' & g'^2 \end{pmatrix} \frac{f_\pi^2}{4} \quad (W^+, W^-, W_3, \mathcal{A})$$

same structure as standard EW theory.

Diagonalize: $M_{W^\pm}^2 = g^2 f_\pi^2 / 4$, $M_Z^2 = (g^2 + g'^2) f_\pi^2 / 4$, $M_A^2 = 0$, so

$$\frac{M_Z^2}{M_{W^\pm}^2} = \frac{(g^2 + g'^2)}{g^2} = \frac{1}{\cos^2 \theta_W}$$

Massless pions disappear from physical spectrum, to become longitudinal components of weak bosons. $M_W \approx 30 \text{ MeV}/c^2$ No fermion masses ...

Origin of fermion masses: quarks & charged leptons

$$\mathcal{L}_{\text{Yukawa}} = -\zeta_d^{ij}(\bar{L}_i\phi)d_{Rj} - \zeta_u^{ij}(\bar{L}_i\bar{\phi})u_{Rj} + \text{h.c.} ,$$

$\zeta_{u,d}$: 3×3 complex matrices i, j : generation indices
 L_i : LH quark doublets u_{Rj}, d_{Rj} : RH quark singlets

$$\text{Mass eigenstates: } \zeta_f^{\text{diag}} = U_L^f \zeta_f U_R^{f\dagger}$$

$f = u, d$: up-like or down-like quarks; $U_{L,R}^f$: unitary matrices

$$\text{Define } \mathbf{u}_L = (u_L, c_L, t_L) \quad \mathbf{d}_L = (d_L, s_L, b_L)$$

$$\mathcal{L}_{\text{CC}}^{(q)} = -\frac{g}{\sqrt{2}}\bar{\mathbf{u}}_L \gamma^\mu \mathbf{V} \mathbf{d}_L W_\mu^+ + \text{h.c.}$$

\mathbf{V} : quark mixing (Cabibbo–Kobayashi–Maskawa) matrix

$$V \equiv U_L^u U_L^{d\dagger} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

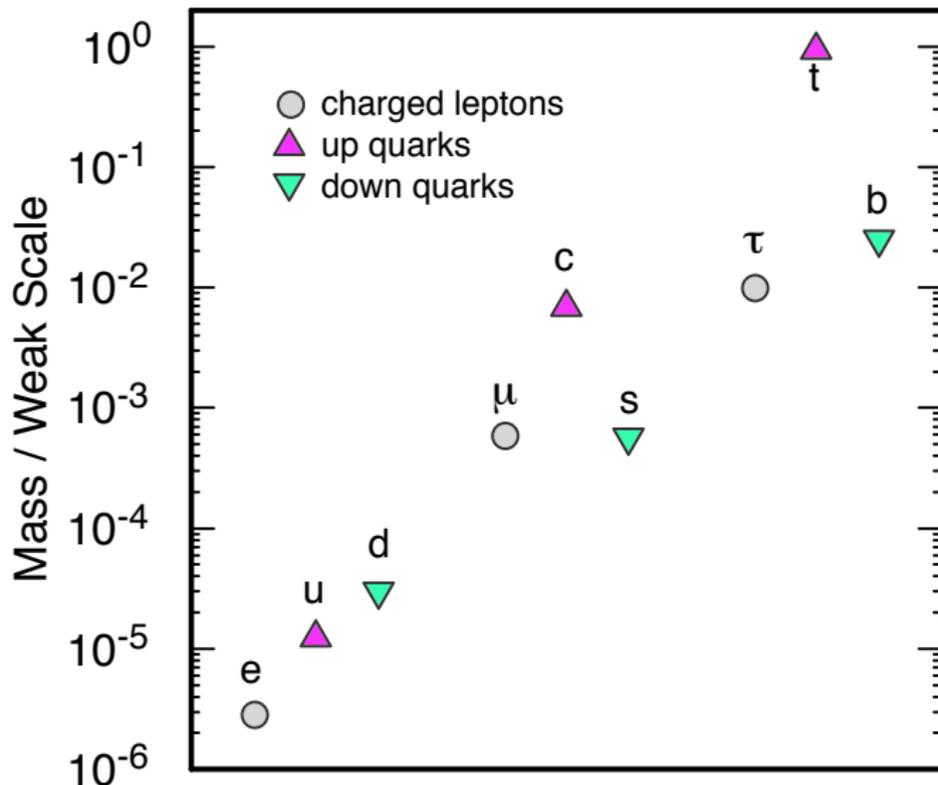
Experiment tells us ...

$$|V| \equiv \begin{pmatrix} |V_{ud}| & |V_{us}| & |V_{ub}| \\ |V_{cd}| & |V_{cs}| & |V_{cb}| \\ |V_{td}| & |V_{ts}| & |V_{tb}| \end{pmatrix} = \begin{pmatrix} 0.974 & 0.227 & 0.004 \\ 0.227 & 0.973 & 0.042 \\ 0.008 & 0.042 & 0.999 \end{pmatrix}$$

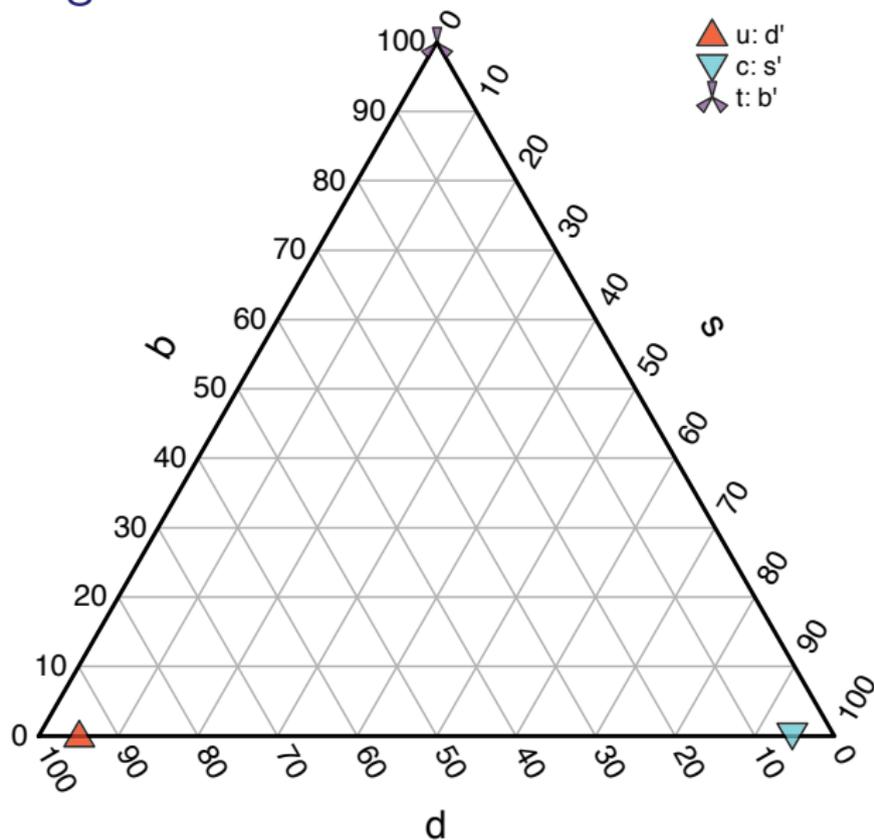
The Higgs scalar is the only element of the standard model that distinguishes among the generations.

Veltman: *It knows something we do not know.*

Yukawa couplings (mass eigenstates) ζ_f^{diag}



Quark Mixing



Components: $|V_{u\alpha}|^2$, etc.

Origin of fermion masses: neutrinos

Define $\nu = (\nu_1, \nu_2, \nu_3)$ $\ell_L = (e_L, \mu_L, \tau_L)$

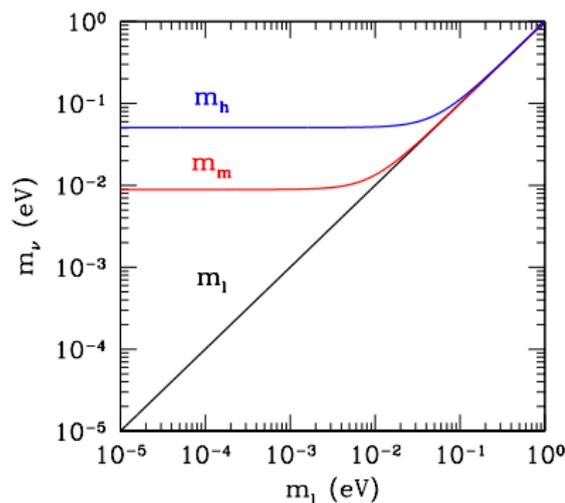
$$\mathcal{L}_{\text{CC}}^{(q)} = -\frac{g}{\sqrt{2}} \bar{\nu} \gamma^\mu \nu^\dagger \ell_L W_\mu^+ + \text{h.c.},$$

\mathcal{V} : ν mixing (Pontecorvo–Maki–Nakagawa–Sakata) matrix

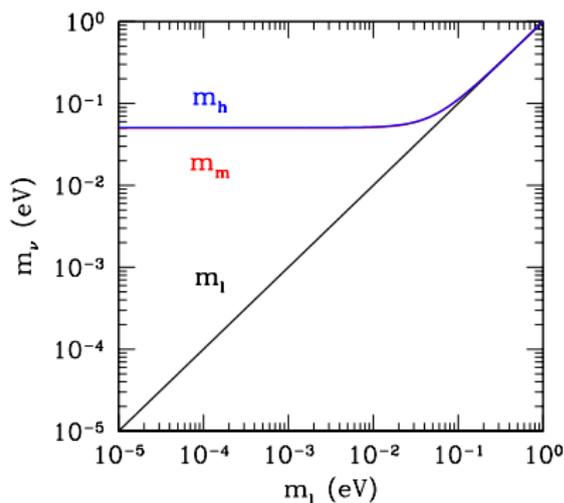
$$\mathcal{V} = \begin{pmatrix} \mathcal{V}_{e1} & \mathcal{V}_{e2} & \mathcal{V}_{e3} \\ \mathcal{V}_{\mu1} & \mathcal{V}_{\mu2} & \mathcal{V}_{\mu3} \\ \mathcal{V}_{\tau1} & \mathcal{V}_{\tau2} & \mathcal{V}_{\tau3} \end{pmatrix}$$

Convention: ν_1, ν_2 : solar pair, $m_1 < m_2$
 ν_3 separated by Δm_{atm}^2 ; above or below?

Absolute scale of neutrino masses is not yet known



Normal spectrum



Inverted spectrum

$$m_2^2 - m_1^2 = \Delta m_\odot^2 = 7.9 \times 10^{-5} \text{ eV}^2 \quad m_3^2 - m_1^2 = \Delta m_{\text{atm}}^2 = 2.5 \times 10^{-3} \text{ eV}^2$$

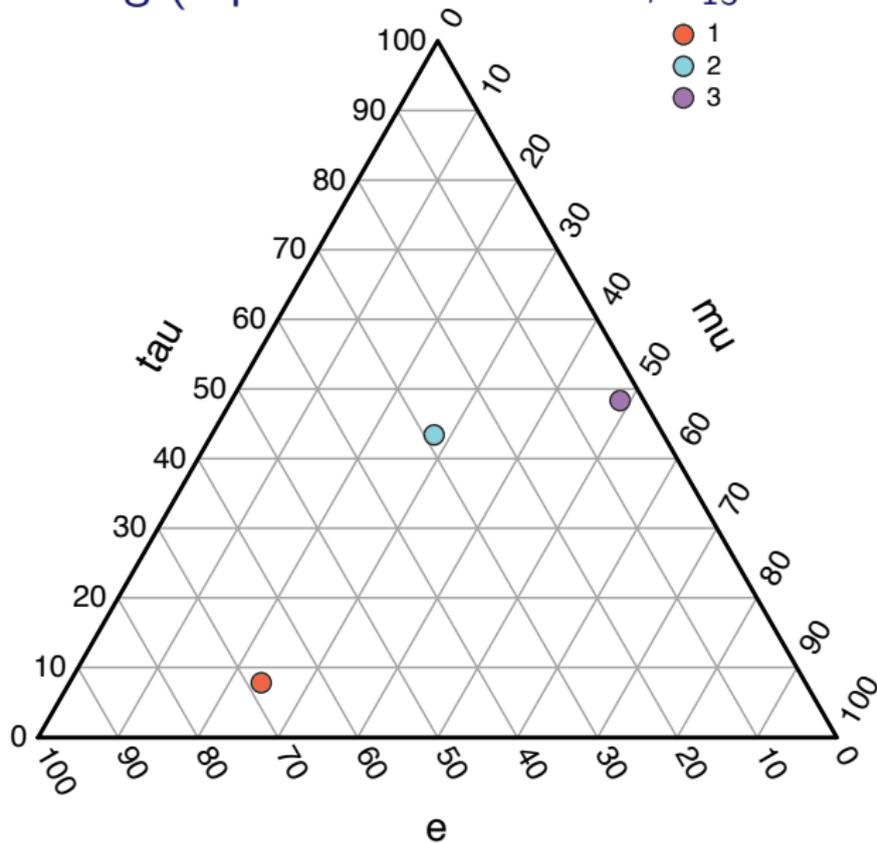
Astronomical data: $\sum_i m_{\nu_i} < 0.62 \text{ eV}$

If Dirac masses, Yukawa couplings $\lesssim 10^{-11}$

Experiment tells us ...

$$|\mathcal{V}| = \begin{pmatrix} 0.79 - 0.88 & 0.47 - 0.61 & < 0.20 \\ 0.19 - 0.52 & 0.42 - 0.73 & 0.58 - 0.82 \\ 0.20 - 0.53 & 0.44 - 0.74 & 0.56 - 0.81 \end{pmatrix}$$

Neutrino Mixing (representative values, $\theta_{13} = 10^\circ$)



Application to cosmic ν : Barenboim & CQ, *Phys. Rev. D***67**, 073024 (2003)

Varieties of neutrino mass: Dirac mass

Chiral decomposition of Dirac spinor:

$$\psi = \frac{1}{2}(1 - \gamma_5)\psi + \frac{1}{2}(1 + \gamma_5)\psi \equiv \psi_L + \psi_R$$

Dirac mass connects LH, RH components of *same field*

$$\mathcal{L}_D = -D(\bar{\psi}_L\psi_R + \bar{\psi}_R\psi_L) = -D\bar{\psi}\psi$$

$$\implies \text{mass eigenstate } \psi = \psi_L + \psi_R$$

*(invariant under global phase rotation $\nu \rightarrow e^{i\theta}\nu$,
 $\ell \rightarrow e^{i\theta}\ell$, so that lepton number is conserved)*

Varieties of neutrino mass: Dirac mass

Add RH neutrino N_R to the standard-model spectrum

N_R : $SU(2)_L$ singlet with $Y = 0$, so **sterile**

$$\mathcal{L}_D^{(\nu)} = -\zeta_\nu [(\bar{L}_\ell \bar{\phi}) N_R + \bar{N}_R (\bar{\phi}^\dagger L_\ell)] \rightarrow -m_D (\bar{\nu}_L N_R + \bar{N}_R \nu_L)$$

$$m_D = \zeta_\nu v / \sqrt{2}$$

Some argue that $\zeta_\nu \lesssim 10^{-11}$ is unnatural, while the range $\zeta_t \approx 1$ to $\zeta_e \approx \text{few} \times 10^{-6}$ is merely puzzling. All Dirac masses involve physics beyond the standard model.

Varieties of neutrino mass: Majorana mass

Neutrinos: no color, no Q : their own antiparticles?

Majorana² fermions

Charge conjugate of RH field is LH: $\psi_L^c \equiv (\psi^c)_L = (\psi_R)^c$

Majorana joins LH, RH components of *conjugate fields*

$$-\mathcal{L}_{MA} = A(\bar{\nu}_R^c \nu_L + \bar{\nu}_L \nu_R^c) = A\bar{\chi}\chi$$

$$-\mathcal{L}_{MB} = B(\bar{\nu}_L^c \nu_R + \bar{\nu}_R \nu_L^c) = B\bar{\omega}\omega$$

for which the mass eigenstates are

$$\chi \equiv \nu_L + \nu_R^c = \chi^c = \nu_L + (\nu_L)^c$$

$$\omega \equiv \nu_R + \nu_L^c = \omega^c = \nu_R + (\nu_R)^c$$

²Escapist Literature: Ettore Majorana vanished without a trace from a ferry between Sicily and the Italian mainland. Leonardo Sciascia has written a fictional account that Majoranas colleagues denounced as scurrilous fantasy. Italian original: *La Scomparsa di Majorana*, English translation: *The Moro Affair and the Mystery of Majorana*.

Lepton number violation

Majorana ν : no conserved additive quantum number

\mathcal{L}_M violates lepton number by two units

\Rightarrow Majorana ν can mediate $\beta\beta_{0\nu}$ decays

$$(Z, A) \rightarrow (Z + 2, A) + e^- + e^-$$

Detecting $\beta\beta_{0\nu}$ would offer decisive evidence for Majorana nature of ν

Active ν_L mass generated by $l = 1$ Higgs with vev or effective operator containing two $l = \frac{1}{2}$ Higgs combined to transform as $l = 1$.

Assessment

$SU(2)_L \otimes U(1)_Y$: 25 years of confirmations

- neutral currents;
- W^\pm, Z^0
- charm

(+ experimental guidance)

- τ, ν_τ
- b, t

+ experimental surprises

- narrowness of ψ, ψ'
- long B lifetime; large $B^0-\bar{B}^0$ mixing
- large $B^0-\bar{B}^0$ mixing
- heavy top
- neutrino oscillations

10 years precise measurements: no significant deviations

Quantum corrections tested at $\pm 10^{-3}$

No “new physics” ... yet!

Theory tested at distances from 10^{-17} cm to $\sim 10^{22}$ cm

origin Coulomb's law (tabletop experiments)

smaller $\left\{ \begin{array}{l} \text{Atomic physics} \rightarrow \text{QED} \\ \text{high-energy expts.} \rightarrow \text{EW theory} \end{array} \right.$

larger $M_\gamma \approx 0$ in planetary ... measurements

Is EW theory true? Is it complete ??

Challenge: Understanding the Everyday (bis)

What would the world be like, without a (Higgs) mechanism to hide electroweak symmetry and give masses to the quarks and leptons?

*Consider the effects of **all** the $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$ interactions!*

With no Higgs mechanism . . .

- Quarks and leptons would remain massless
- QCD would confine the quarks in color-singlet hadrons
- *N mass little changed*, but p outweighs n
- QCD breaks EW to EM, gives $(1/2500 \times \text{observed})$ masses to W, Z , so weak-isospin force doesn't confine
- **Rapid!** β -decay \Rightarrow lightest nucleus is n ; no H atom
- Some light elements in BBN (?), but ∞ Bohr radius
- No atoms (as we know them) means no chemistry, no stable composite structures like solids and liquids

. . . the character of the physical world would be profoundly changed

Parameters of the Standard Model

- 3 coupling parameters $\alpha_s, \alpha_{EM}, \sin^2 \theta_W$
 - 2 parameters of the Higgs potential
 - 1 vacuum phase (QCD)
 - 6 quark masses
 - 3 quark mixing angles
 - 1 CP-violating phase
 - 3 charged-lepton masses
 - 3 neutrino masses
 - 3 leptonic mixing angles
 - 1 leptonic CP-violating phase (+ Majorana ...)
-

26⁺ arbitrary parameters

parameter count not improved by unification

The EW scale and beyond

EWSB scale, $v = (G_F \sqrt{2})^{-\frac{1}{2}} \approx 246$ GeV, sets

$$M_W^2 = g^2 v^2 / 2 \quad M_Z^2 = M_W^2 / \cos^2 \theta_W$$

But it is not the only scale of physical interest

quasi-certain: $M_{\text{Planck}} = 1.22 \times 10^{19}$ GeV

probable: $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$ unification scale $\sim 10^{15-16}$ GeV

somewhere: flavor scale

How to keep the distant scales from mixing in the face of quantum corrections?

OR

How to stabilize the mass of the Higgs boson on the electroweak scale?

OR

Why is the electroweak scale small?

“The hierarchy problem”

Loop integrals are potentially divergent

$$m^2(p^2) = m^2(\Lambda^2) + Cg^2 \int_{p^2}^{\Lambda^2} dk^2 + \dots$$

Λ : reference scale at which m^2 is known

g : coupling constant of the theory

C : coefficient calculable in specific theory

For mass shifts induced by radiative corrections to remain under control (not greatly exceed the value measured on the laboratory scale), *either*

- Λ must be small, *or*
- New Physics must intervene to cut off integral

But natural reference scale for Λ is

$$\Lambda \approx M_{\text{Planck}} = \left(\frac{\hbar c}{G_{\text{Newton}}} \right)^{1/2} \approx 1.22 \times 10^{19} \text{ GeV}$$

for $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$

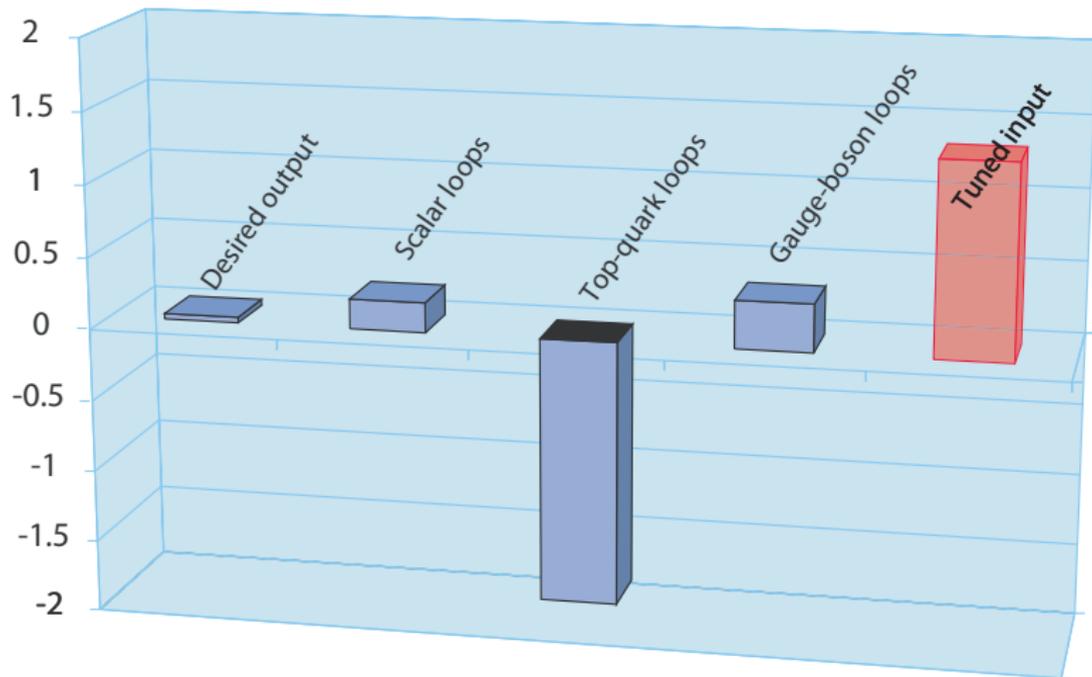
or

$$\Lambda \approx M_U \approx 10^{15} - 10^{16} \text{ GeV} \quad \text{for unified theory}$$

$$\text{Both} \gg v/\sqrt{2} \approx 175 \text{ GeV} \quad \Rightarrow$$

New Physics at $E \lesssim 1 \text{ TeV}$

δM_H^2 for 5-TeV cutoff



$$\delta M_H^2 = \frac{G_F \Lambda^2}{4\pi^2 \sqrt{2}} (6M_W^2 + 3M_Z^2 + M_H^2 - 12m_t^2)$$

Only a few distinct scenarios ...

- Supersymmetry: balance contributions of fermion loops (-1) and boson loops ($+1$)
Exact supersymmetry,

$$\sum_{\substack{i=\text{fermions} \\ +\text{bosons}}} C_i \int dk^2 = 0$$

Broken supersymmetry, shifts acceptably small if superpartner mass splittings are not too large

$$g^2 \Delta M^2 \text{ "small enough"} \Rightarrow \tilde{M} \lesssim 1 \text{ TeV}/c^2$$

Only a few distinct scenarios . . .

- Composite scalars (technicolor): New physics arises on scale of composite Higgs-boson binding,

$$\Lambda_{\text{TC}} \simeq O(1 \text{ TeV})$$

“Form factor” cuts effective range of integration

Only a few distinct scenarios . . .

- Strongly interacting gauge sector: WW resonances, multiple W production, probably scalar bound state “quasiHiggs” with $M < 1$ TeV
- Spontaneously broken global symmetries \rightsquigarrow pseudo-Nambu–Goldstone bosons
 - ▶ Higgs boson would be massless if Nambu–Goldstone boson protected against large quantum corrections
 - ▶ Global symmetry must be large enough that left-over NGBs remain after some provide longitudinal components of massive gauge bosons
 - ▶ Explicit symmetry breaking needed for $M_H \neq 0$
Extra gauge bosons ($M_{W'} \approx 4\pi M_H$), enlarged fermion multiplets cancel quadratic divergences in δM_H^2

“Little Higgs” effective theories, $\Lambda \approx (4\pi)^2 M_H$

Only a few distinct scenarios . . .

- Or maybe the problem is with (our understanding of) *gravity*, not with the electroweak theory?

Large extra dimensions

Arkani-Hamed, Dimopoulos, Dvali, *Phys. Lett.* **429**, 263 (1998)

Universal extra dimensions

Appelquist, Cheng, Dobrescu, *Phys. Rev. D* **64**, 035002 (2001)

Warped extra dimensions

Randall & Sundrum, *Phys. Rev. Lett.* **83**, 3370, 4690 (1999)

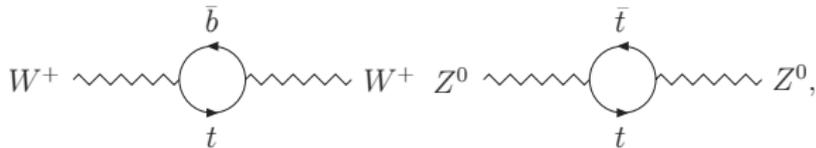
Higgs boson: the missing element of electroweak theory



Experimental clues to the Higgs-boson mass

Sensitivity of EW observables to m_t gave early indications for massive top

Quantum corrections to SM predictions for M_W and M_Z arise from different quark loops



$$\dots \text{alter the link } \underbrace{M_W^2}_{(80.398 \pm 0.025 \text{ GeV})^2} = \underbrace{M_Z^2 (1 - \sin^2 \theta_W)}_{(80.939 \text{ GeV})^2} (1 - \Delta\rho)$$

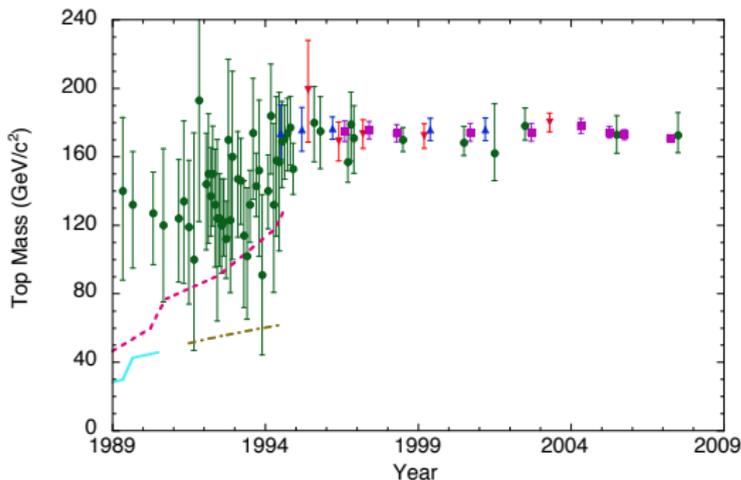
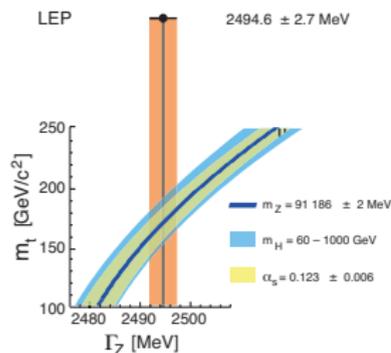
$$\text{where } \Delta\rho \approx \Delta\rho^{(\text{quarks})} = 3G_F m_t^2 / 8\pi^2 \sqrt{2}$$

Strong dependence on m_t^2 accounts for precision of m_t estimates derived from EW observables

Tevatron: $\delta m_t / m_t \approx 1.28\%$. . . Look beyond quark loops to next most important quantum corrections: Higgs-boson effects

Global fits to precision EW measurements

- precision improves with time / calculations improve with time



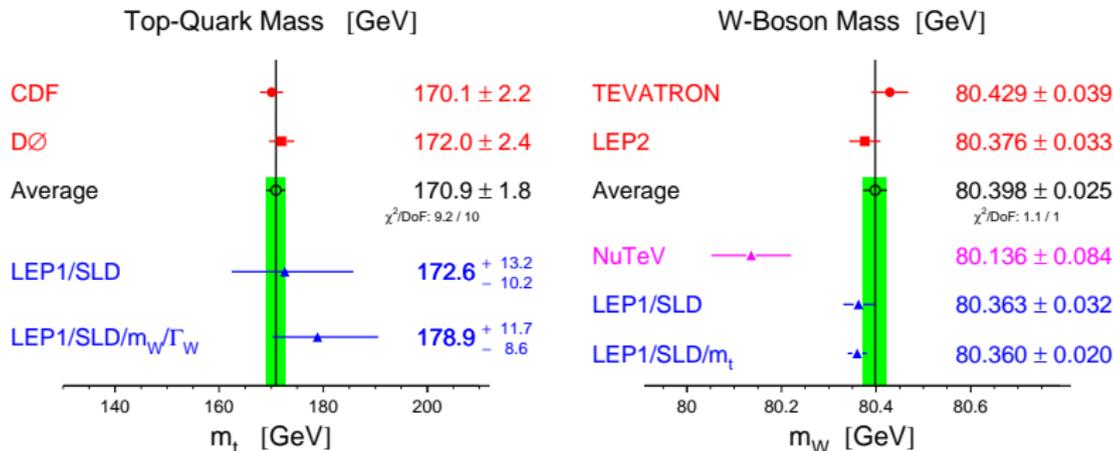
$$11.94, \text{LEPEWWG: } m_t = 178 \pm 11_{-19}^{+18} \text{ GeV}/c^2$$

$$\text{Direct measurements: } m_t = 170.9 \pm 1.8 \text{ GeV}/c^2$$

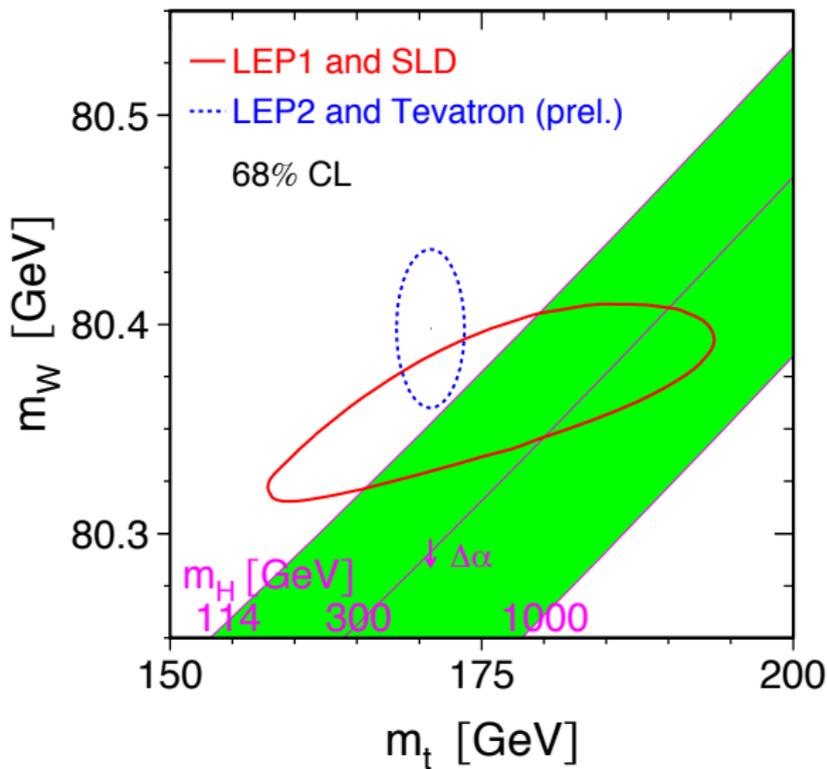
H quantum corrections smaller than t corrections, exhibit more subtle dependence on M_H than the m_t^2 dependence of the top-quark corrections

$$\Delta\rho^{(\text{Higgs})} = C \cdot \ln\left(\frac{M_H}{v}\right)$$

M_Z known to 23 ppm, m_t and M_W well measured

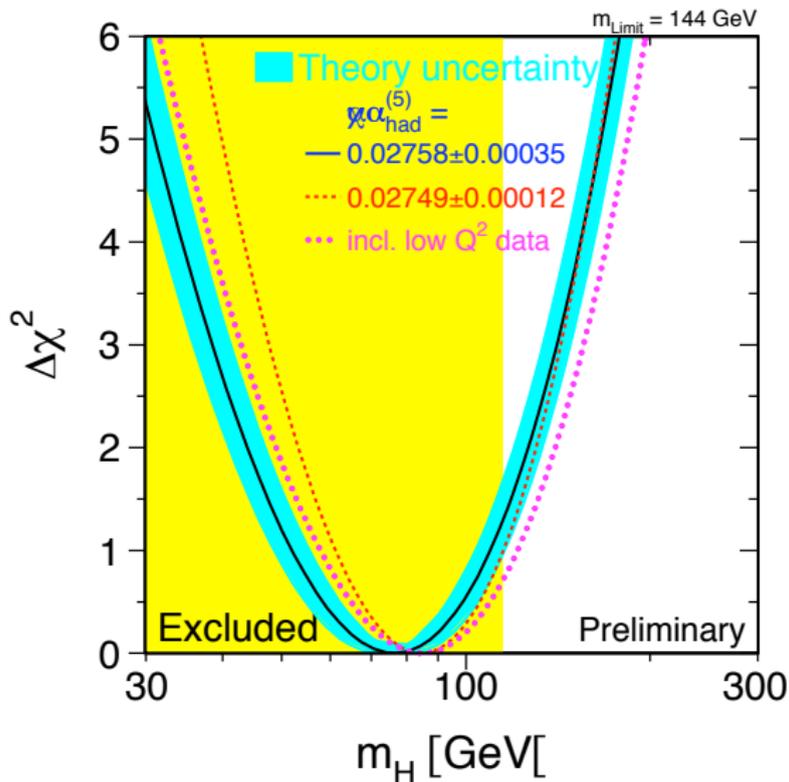


... so examine dependence of M_W upon m_t and M_H



Direct, indirect determinations agree reasonably
 Both favor a light Higgs boson, ... *within framework of SM analysis.*

Fit to a universe of data



Standard-Model $M_H \lesssim 182 \text{ GeV}$ at 95% CL

Fit to a universe of data . . .

- *Within SM*, LEP EWWG deduce a 95% CL upper limit, $M_H \lesssim 182 \text{ GeV}/c^2$
- Direct searches at LEP $\Rightarrow M_H > 114.4 \text{ GeV}/c^2$, excluding much of the favored region
- Either the Higgs boson is just around the corner, or SM analysis is misleading

Things will soon be popping!

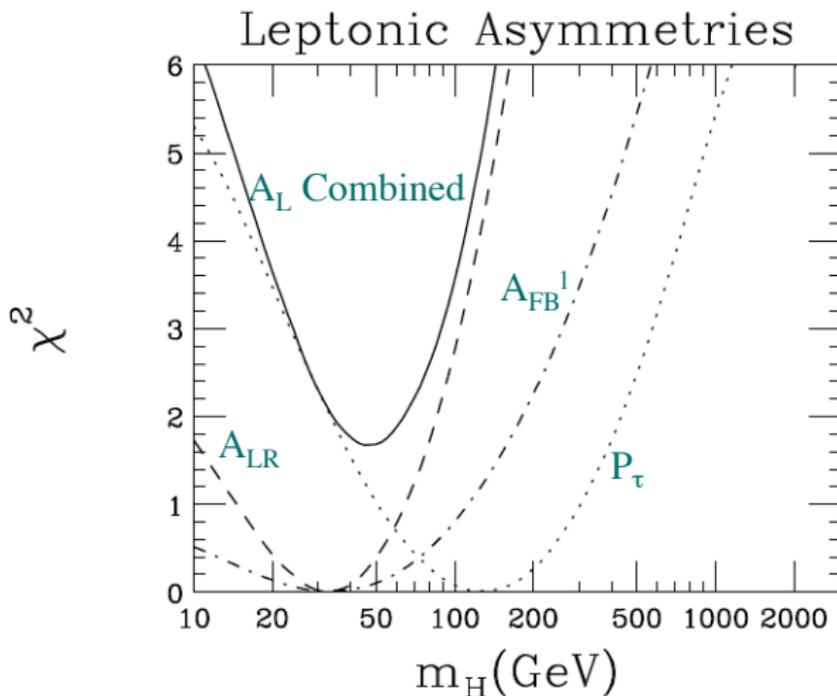
A Cautionary Note

- A_{FB}^b , which exerts the greatest “pull” on the global fit [slide 83], is most responsible for raising M_H above the range excluded by direct searches [slide 122].
- Leptonic and hadronic observables point to different best-fit values of M_H
- Many subtleties in experimental and theoretical analyses

M. Chanowitz, *Phys. Rev. Lett.* **87**, 231802 (2001); *Phys. Rev.* **D66**, 073002 (2002); hep-ph/0304199;
<http://phyweb.lbl.gov/~chanowitz/rpm-10-06.pdf>

Introduction to global analyses: J. L. Rosner, hep-ph/0108195;
hep-ph/0206176

χ^2 Distributions: Leptonic Asymmetries

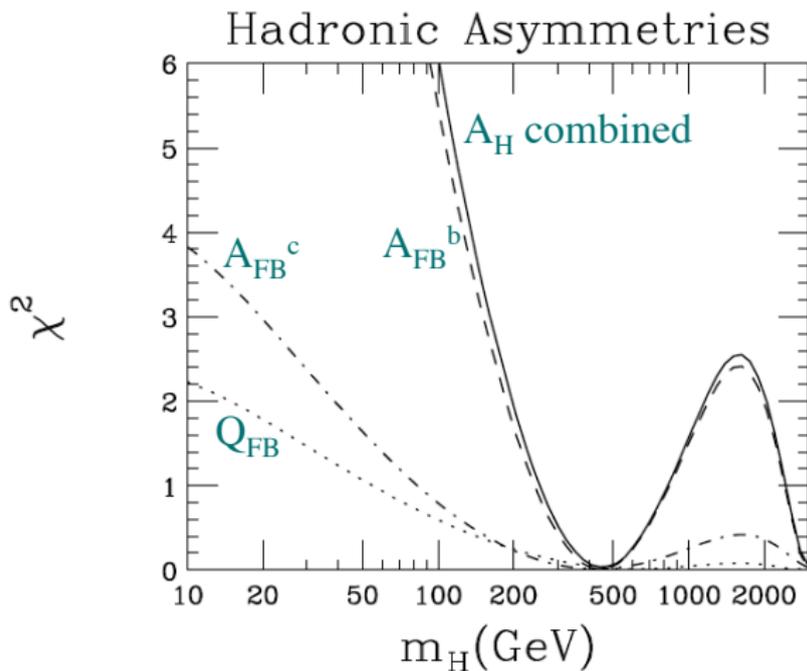


M. Chanowitz

RPM 10/26/06

20

χ^2 Distributions: Hadronic Asymmetries



M. Chanowitz

RPM 10/26/06

21

- Tevatron, LHC measurements will determine m_t within 1 or 2 GeV
 ... and improve δM_W to about 15 MeV
- As the Tevatron's integrated luminosity approaches 10 fb^{-1} , CDF and DØ will explore the region of M_H not excluded by LEP
- ATLAS and CMS will carry on the exploration of the Higgs sector at the LHC;
 could require a few years, at low mass;
 full range accessible, $\gamma\gamma, \ell\ell\nu\nu, b\bar{b}, \ell^+\ell^-\ell^+\ell^-, \ell\nu jj, \tau\tau$ channels.

Electroweak theory confronts experiment

- G. Altarelli and M. Grünewald, “Precision Electroweak Tests of the SM,” hep-ph/0404165.
- F. Teubert, “Precision tests of the electroweak interactions,” *Int. J. Mod. Phys. A* **20**, 5174 (2005).
- S. de Jong, “Tests of the Electroweak Sector of the Standard Model,” *PoS HEP2005*, 397 (2006) [hep-ph/0512043].

Details of Electroweak Theory . . .

For more, see my

- “Spontaneous symmetry breaking as a basis of particle mass,” *Rep. Prog. Phys.* **70**, 1019 (2007), arXiv:0704.2232.
- “The Electroweak Theory,” in *Flavor Physics for the Millennium: TASI 2000*, edited by Jonathan L. Rosner (World Scientific, Singapore, 2001), pp. 367; hep-ph/0204104.
- *Gauge Theories of the Strong, Weak, and Electromagnetic Interactions* (Westview Press, 1997)
http://www.perseusbooksgroup.com/perseus/book_detail.jsp?isbn=0201328321

Appendix: More on the Higgs boson

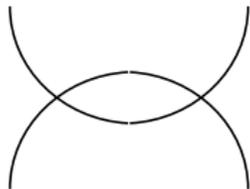
Bounding M_H from above . . .

Triviality of scalar field theory

- Only *noninteracting* scalar field theories make sense on all energy scales
- Quantum field theory vacuum is a dielectric medium that screens charge
- \Rightarrow *effective charge* is a function of the distance or, equivalently, of the energy scale

running coupling constant

In $\lambda\phi^4$ theory, calculate variation of coupling constant λ in perturbation theory by summing bubble graphs



$\lambda(\mu)$ is related to a higher scale Λ by

$$\frac{1}{\lambda(\mu)} = \frac{1}{\lambda(\Lambda)} + \frac{3}{2\pi^2} \log(\Lambda/\mu)$$

(Perturbation theory reliable only when λ is small, lattice field theory treats strong-coupling regime)

For stable Higgs potential (*i.e.*, for vacuum energy not to race off to $-\infty$), *require* $\lambda(\Lambda) \geq 0$

Rewrite RGE as an inequality

$$\frac{1}{\lambda(\mu)} \geq \frac{3}{2\pi^2} \log(\Lambda/\mu)$$

... implies an *upper bound*

$$\lambda(\mu) \leq 2\pi^2/3 \log(\Lambda/\mu)$$

If we require the theory to make sense to arbitrarily high energies—or short distances—then we must take the limit $\Lambda \rightarrow \infty$ while holding μ fixed at some reasonable physical scale. In this limit, the **bound** forces $\lambda(\mu)$ to zero.

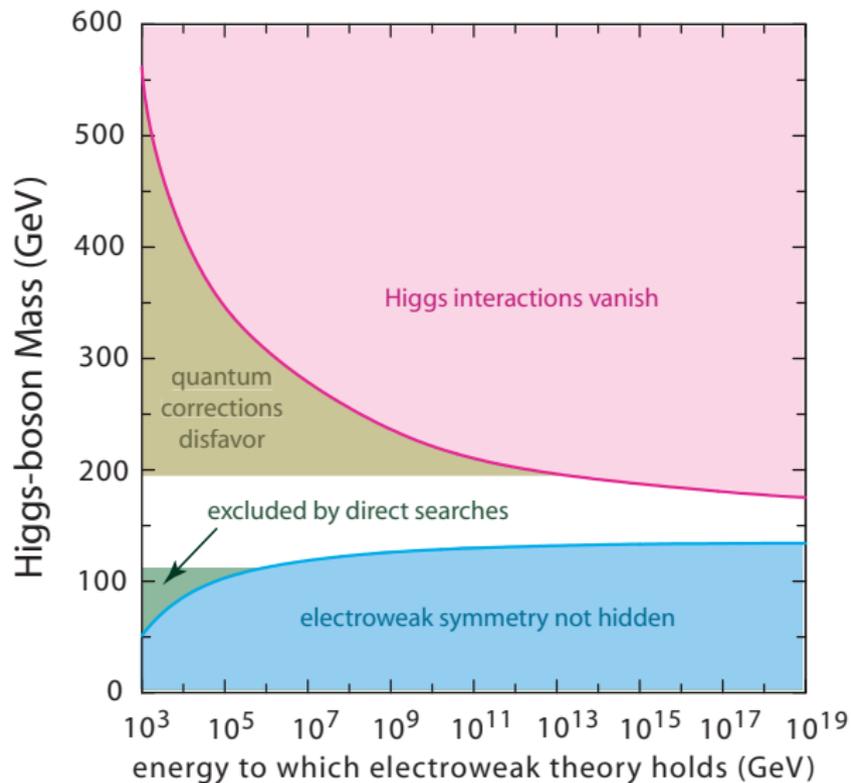
→ free field theory “trivial”

Rewrite as bound on M_H :

$$\Lambda \leq \mu \exp\left(\frac{2\pi^2}{3\lambda(\mu)}\right)$$

Choose $\mu = M_H$, and recall $M_H^2 = 2\lambda(M_H)v^2$

$$\Lambda \leq M_H \exp\left(4\pi^2 v^2 / 3M_H^2\right)$$



Moral: For any M_H , there is a *maximum energy scale* Λ^* at which the theory ceases to make sense.

The description of the Higgs boson as an elementary scalar is at best an effective theory, valid over a finite range of energies

Perturbative analysis breaks down when $M_H \rightarrow 1 \text{ TeV}/c^2$ and interactions become strong

Lattice analyses $\implies M_H \lesssim 710 \pm 60 \text{ GeV}/c^2$ if theory describes physics to a few percent up to a few TeV

If $M_H \rightarrow 1 \text{ TeV}$ EW theory lives on brink of instability

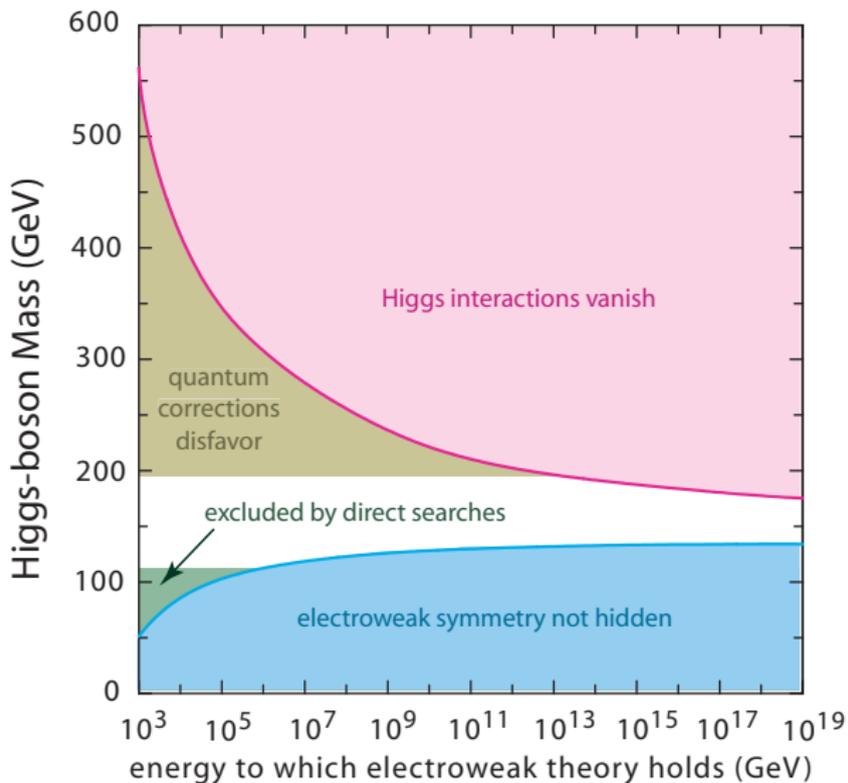
Lower bound by requiring EWSB vacuum $V(v) < V(0)$

Requiring that $\langle \phi \rangle_0 \neq 0$ be an absolute minimum of the one-loop potential up to a scale Λ yields the vacuum-stability condition ... (for $m_t \lesssim M_W$)

$$M_H^2 > \frac{3G_F\sqrt{2}}{8\pi^2} (2M_W^4 + M_Z^4 - 4m_t^4) \log(\Lambda^2/v^2)$$

(No illuminating analytic form for heavy m_t)

If the Higgs boson is relatively light (which would require explanation) then the theory can be self-consistent up to very high energies



If EW theory is to make sense all the way up to a unification scale $\Lambda^* = 10^{16}$ GeV, then $134 \text{ GeV}/c^2 \lesssim M_H \lesssim 177 \text{ GeV}$

Higgs-Boson Properties

$$\Gamma(H \rightarrow f\bar{f}) = \frac{G_F m_f^2 M_H}{4\pi\sqrt{2}} \cdot N_c \cdot \left(1 - \frac{4m_f^2}{M_H^2}\right)^{3/2}$$

$\propto M_H$ in the limit of large Higgs mass; $\propto \beta^3$ for scalar

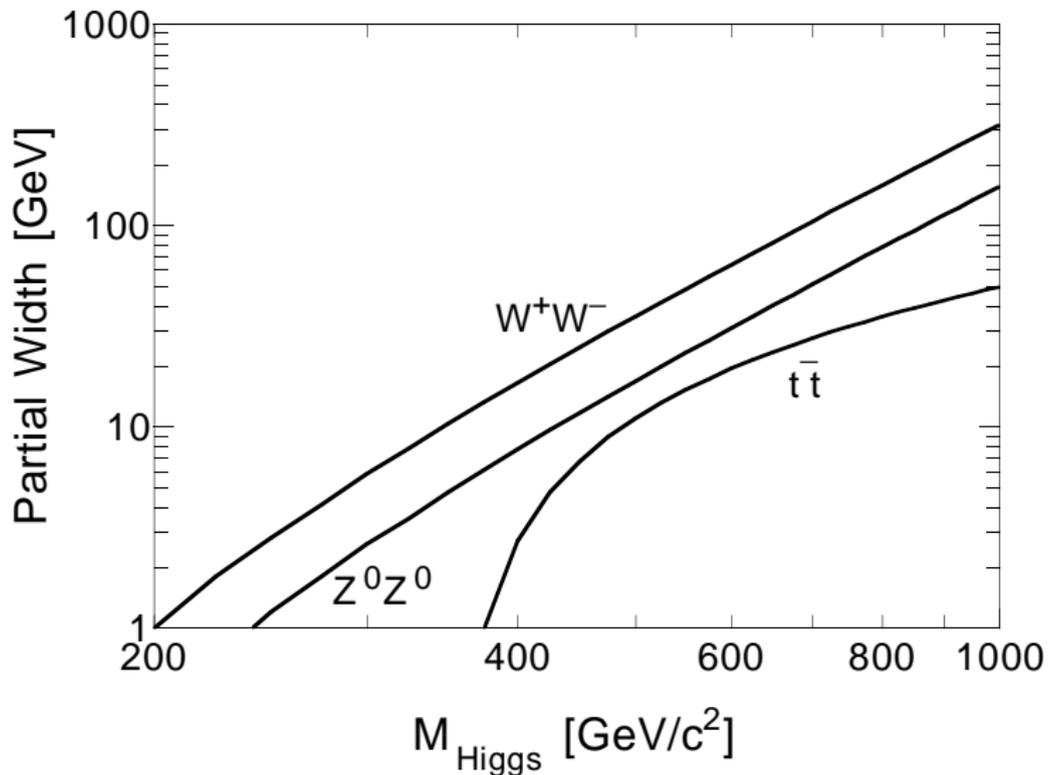
$$\Gamma(H \rightarrow W^+W^-) = \frac{G_F M_H^3}{32\pi\sqrt{2}} (1-x)^{1/2} (4-4x+3x^2) \quad x \equiv 4M_W^2/M_H^2$$

$$\Gamma(H \rightarrow Z^0Z^0) = \frac{G_F M_H^3}{64\pi\sqrt{2}} (1-x')^{1/2} (4-4x'+3x'^2) \quad x' \equiv 4M_Z^2/M_H^2$$

asymptotically $\propto M_H^3$ and $\frac{1}{2}M_H^3$, respectively $\left(\frac{1}{2}\right.$ from weak isospin)

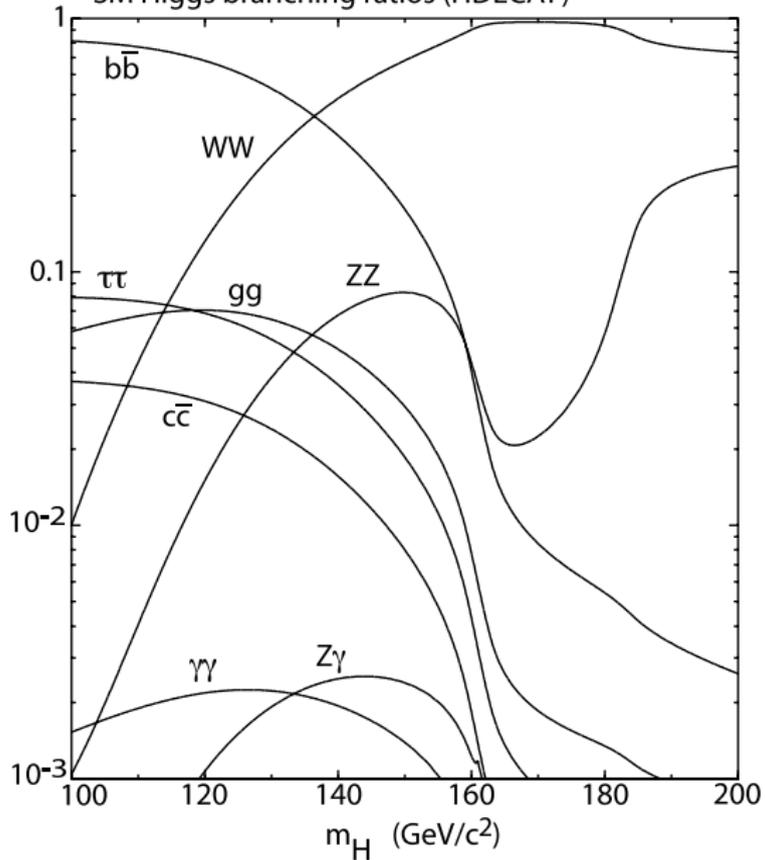
$2x^2$ and $2x'^2$ terms \Leftrightarrow decays into transverse gauge bosons

Dominant decays for large M_H : pairs of longitudinal weak bosons

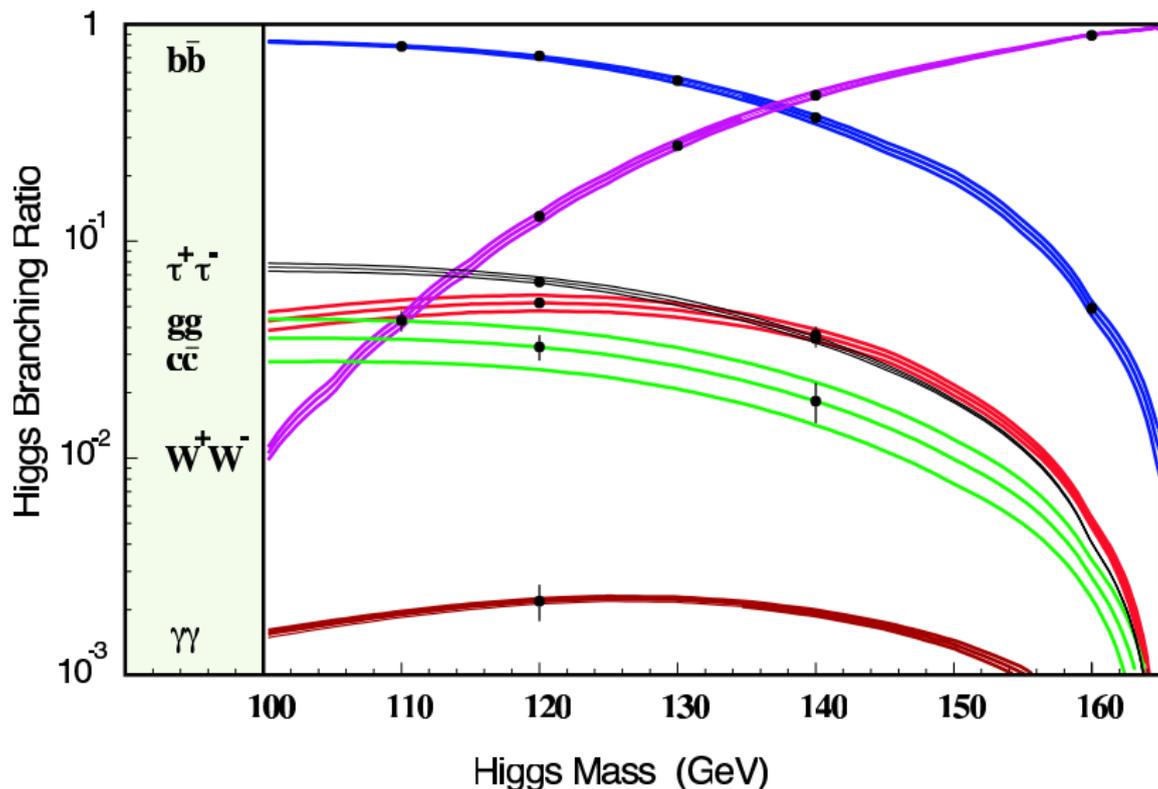


For $M_H \rightarrow 1$ TeV, Higgs boson is *ephemeral*: $\Gamma_H \rightarrow M_H$.

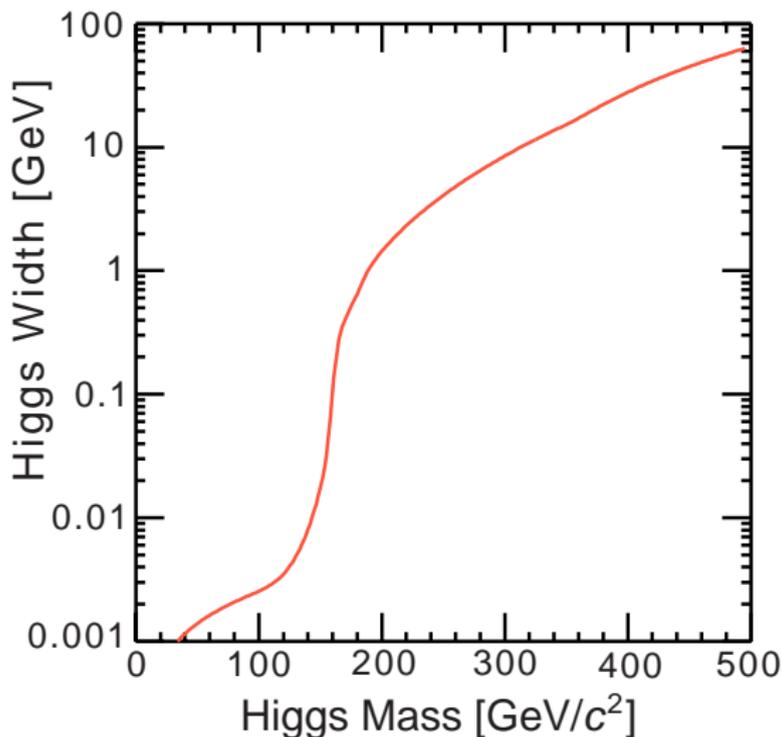
SM Higgs branching ratios (HDECAY)



ILC would measure light Higgs-boson couplings precisely



Points: 500 fb^{-1} @ 350 GeV Bands: theory uncertainty (m_b)



Below W^+W^- threshold, $\Gamma_H \lesssim 1$ GeV

Far above W^+W^- threshold, $\Gamma_H \propto M_H^3$