Fermilab

Tutorial July 5, Boris Kayser, V Siemmer School

1) PPT Tutorial 070705.ppt showing there is a matter effect, and CP, even when $\overline{\nu}_i = \nu_i$.

2) ~~CP~~

$$CP\,|\,f(\vec{p}, h)\rangle = \eta_{CP}^{(f)}\,|\,\overline{f}(-\vec{p}, -h)\rangle$$

(h: helicity)

CP invariance: $(CP)^{-1} H (CP) = H$

$(CP)^{-1} = (CP)^{\dagger}$

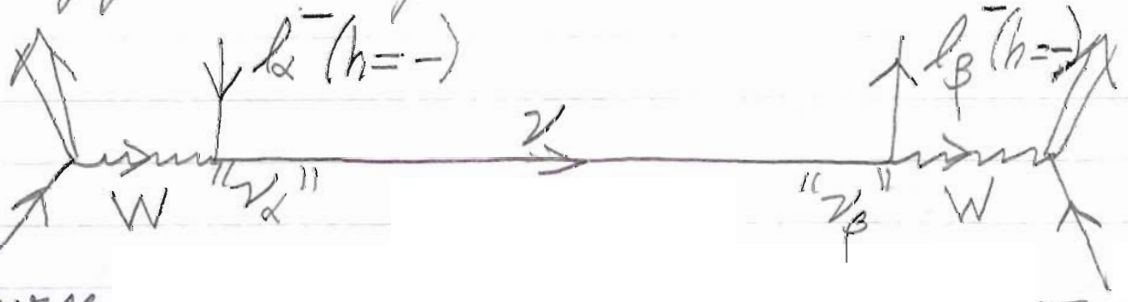
Then

$$\langle f|H|i\rangle = \langle f|(CP)^{\dagger} H (CP)|i\rangle$$

$$= \langle CP[f]|\,H\,|CP[i]\rangle$$

Amp[Process] = Amp[CP-mirror-image process]

Suppose the process is neutrino oscillation.

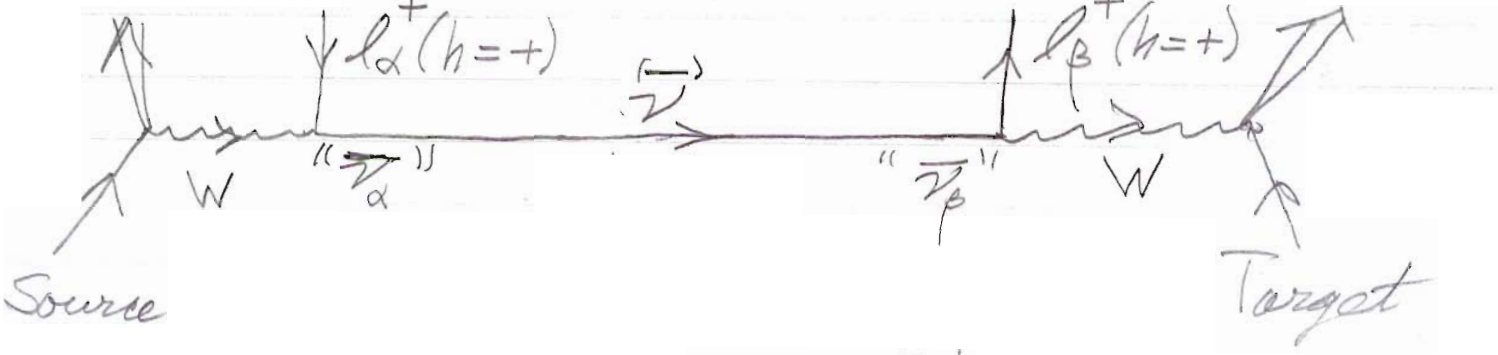


$$Amp = \sum_i U_{\alpha i}^{*} U_{\beta i}\, e^{-i m_i^2 \frac{L}{2E}}$$ ← from (see next page)

$$\mathcal{L}_{W\ell\nu} = -\frac{g}{\sqrt{2}} \sum_{\substack{\alpha = e,\mu,\tau \\ i = 1,2,3}} \left(\bar{\ell}_{L\alpha} \gamma^\lambda U_{\alpha i} \nu_{Li} W_\lambda^- + \bar{\nu}_{Li} \gamma^\lambda U_{\alpha i}^* \ell_{L\alpha} W_\lambda^+ \right)$$

Do R_π to get the momenta right

The CP-mirror-image process, at least for the leptonic part, is



$$\overline{Amp} = \sum_i U_{\alpha i} U_{\beta i}^* \, e^{-i m_i^2 \frac{L}{2E}}$$

If U is not real,

$$\overline{Amp} \neq Amp$$

and

$$\overline{Amp} \neq Amp^*$$

This results in $P[\text{"}\bar{\nu}_\alpha\text{"} \to \text{"}\bar{\nu}_\beta\text{"}] \neq P[\text{"}\nu_\alpha\text{"} \to \text{"}\nu_\beta\text{"}]$

Notice that, for this inequality, which violates at least CP invariance, to appear, we need to have both CP-odd phases {here, $\arg(U_{\alpha i}^* U_{\beta i})$}

and CP-even ones {here $-\Delta m^2_{ij}\frac{L}{2E}$}.

This is how CP works everywhere: It comes from the interference between terms that differ in their CP-odd phase, and also in their CP-even phase.

Note that the CP-violating difference

$$\Delta_{\alpha\beta} \equiv P(\nu_\alpha \to \nu_\beta) - P(\bar{\nu}_\alpha \to \bar{\nu}_\beta)$$

is completely insensitive to whether neutrinos are Dirac or Majorana particles.

CPT constraints:

$$P(\nu_\alpha \to \nu_\beta) \underset{CPT}{=} P(\bar{\nu}_\beta \to \bar{\nu}_\alpha)$$

In particular,

$$P(\nu_\alpha \to \nu_\alpha) \underset{CPT}{=} P(\bar{\nu}_\alpha \to \bar{\nu}_\alpha)$$

No δ in survival probabilities.

CPT invariance $\Rightarrow$

$$\Delta_{\beta\alpha} = P(\nu_\beta \to \nu_\alpha) - P(\bar{\nu}_\beta \to \bar{\nu}_\alpha)$$

$$\underset{CPT}{=} -\left[P(\nu_\alpha \to \nu_\beta) - P(\bar{\nu}_\alpha \to \bar{\nu}_\beta)\right] = -\Delta_{\alpha\beta}$$

CP is equivalent to T.

Flux conservation: $\sum_{\substack{\beta=\alpha \\ \text{and } \beta\neq\alpha}} P(\overset{(-)}{\nu}_\alpha \to \overset{(-)}{\nu}_\beta) = 1$.

Now suppose there are only the 3 neutrino flavors ν_e, ν_μ, ν_τ. Then there are only 3 independent non-zero CP differences:

$$\Delta_{e\mu}, \Delta_{\mu\tau}, \Delta_{\tau e}$$

Let us compare these. We write

$$P(\nu_\alpha \to \nu_\beta) \equiv P_{\alpha\beta}; \quad P(\bar{\nu}_\alpha \to \bar{\nu}_\beta) \equiv \bar{P}_{\alpha\beta}.$$

We have

$$\Delta_{e\mu} \equiv P_{e\mu} - \bar{P}_{e\mu} = (\cancel{1} - \cancel{P_{ee}} - P_{e\tau}) - (\cancel{1} - \cancel{\bar{P}_{ee}} - \bar{P}_{e\tau})$$

$$= \bar{P}_{e\tau} - P_{e\tau} = P_{\tau e} - \bar{P}_{\tau e} = \Delta_{\tau e}.$$

Also,

$$\Delta_{eu} = -\Delta_{ue} = -(P_{ue} - \bar{P}_{ue})$$
$$= (\cancel{1} - \cancel{P_{uu}} - \bar{P}_{u\tau}) - (\cancel{1} - \cancel{P_{uu}} - P_{u\tau})$$
$$= P_{u\tau} - \bar{P}_{u\tau} = \Delta_{u\tau}$$

$$\therefore \quad \Delta_{eu} = \Delta_{u\tau} = \Delta_{\tau e} .$$

There is only <u>one</u> CP difference.

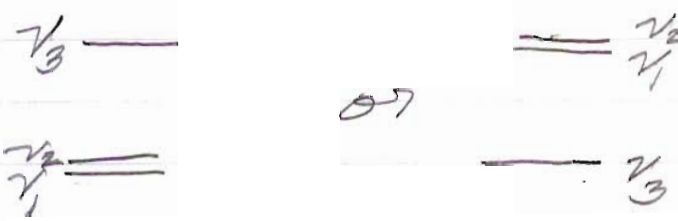
If measurements of the different $\Delta_{\alpha\beta}$ find different answers, there are <u>more</u> than three flavors.

Since ν_τ are hard to work with, the CP difference experiments will look first is Δ_{eu} or Δ_{ue}.

<u>Assuming</u> that the mixing matrix U is unitary it is possible to infer the existence of CP indirectly without actually seeing it. This is because knowing <u>only</u> the $|U_{\alpha i}|$ may tell you that U can't be unitary unless it contains CP phases.

Example (still assuming just 3 flavors):

We know the 3ν spectrum looks like



Suppose we somehow learn that ν_3 is 49% ν_μ and 49% ν_τ, and ν_2 is 33% ν_μ and 33% ν_τ. (These numbers are consistent with the atmospheric and solar oscillation data.) Remembering that $|\langle \nu_\alpha | \nu_i \rangle|^2 = |U_{\alpha i}|^2$, we then have

$$|U|^2 \equiv \begin{bmatrix} |U_{e1}|^2 & |U_{e2}|^2 & |U_{e3}|^2 \\ |U_{\mu 1}|^2 & |U_{\mu 2}|^2 & |U_{\mu 3}|^2 \\ |U_{\tau 1}|^2 & |U_{\tau 2}|^2 & |U_{\tau 3}|^2 \end{bmatrix}$$

$$= \begin{bmatrix} .64 & .34 & .02 \\ .18 & .33 & .49 \\ .18 & .33 & .49 \end{bmatrix}$$

Given

Then, disregarding signs and phases,

$$U = \begin{bmatrix} .800 & .583 & .141 \\ .424 & .574 & .700 \\ .424 & .574 & .700 \end{bmatrix}$$

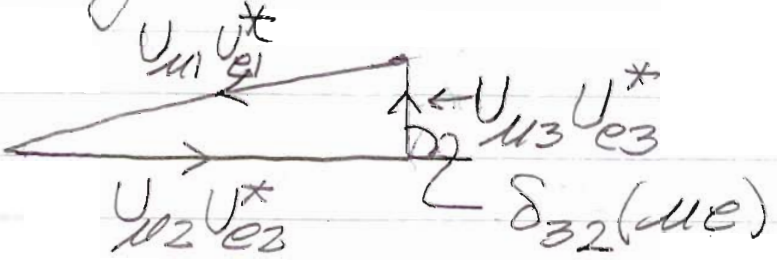
The unitarity constraint expressing the orthogonality of the e and μ rows of U (including signs and phases) is

$$U_{\mu 1} U_{e1}^* + U_{\mu 2} U_{e2}^* + U_{\mu 3} U_{e3}^* = 0$$

In the complex plane, this constraint may be pictured as the statement that its 3 terms are vectors forming the sides of a closed triangle. We have

$|U_{\mu 1} U_{e1}| = .339$, $|U_{\mu 2} U_{e2}| = .335$, $|U_{\mu 3} U_{e3}| = .099$.

Thus, the triangle looks like



There is no way this triangle can close without U containing phases.

A triangle which is a picture of an orthogonality condition coming from the unitarity of the 3×3 U or its quark analogue, the CKM matrix, is known as a unitarity triangle.

Now, we derived in lecture I that for three flavors and $\beta \neq \alpha$,

$$P(\overset{(-)}{\nu}_\alpha \to \overset{(-)}{\nu}_\beta) = 4\{|U_{\alpha 3} U_{\beta 3}|^2 \sin^2\Delta_{31} + |U_{\alpha 2} U_{\beta 2}|^2 \sin^2\Delta_{21}$$

(Added)

$$+ 2|U_{\alpha 3} U_{\beta 3} U_{\alpha 2} U_{\beta 2}| \sin\Delta_{31} \sin\Delta_{21} \cos(\Delta_{32} \pm \delta_{32}^{(\alpha\beta)})\},$$

where $\Delta_{ij} \equiv \Delta m^2_{ij} \frac{L}{4E}$, and

$$\delta_{32}(\alpha\beta) \equiv \arg(U_{\alpha 3} U^*_{\beta 3} U^*_{\alpha 2} U_{\beta 2}).$$

Let us apply this to the channel that will be probed first in CP searches: $\nu_\mu \to \nu_e$. With $\alpha = \mu$, $\beta = e$, we are concerned with

$$\delta_{32}(\mu e) = \arg(U_{\mu 3} U^*_{e 3} U^*_{\mu 2} U_{e 2})$$

Notice that $\delta_{32}(\mu e)$ in our unitarity triangle is not zero, so we will have CP: $P(\overset{(-)}{\nu}_\mu \to \overset{(-)}{\nu}_e) \neq P(\overset{(-)}{\nu}_\mu \to \overset{(-)}{\nu}_e)$. From the magnitudes of the Δ sides, $\delta_{32}(\mu e) = \underline{96^\circ}$.

We have

$$\Delta_{\mu e} \equiv P(\nu_\mu \to \nu_e) - P(\bar{\nu}_\mu \to \bar{\nu}_e) =$$

$$= 16 \underbrace{|U_{\mu 3} U_{e3} U_{\mu 2} U_{e2}| \sin\delta_{32}}_{\text{2 x Area of the Unitarity Triangle}} \underbrace{\sin\Delta_{31} \sin\Delta_{21} \sin\Delta_{32}}_{\text{Kinematical Factors}}$$

The CP-violating $\nu - \bar{\nu}$ difference in oscillation (assuming just 3 flavors) is the Area of the Unitarity Triangle × Kinematical Factors.