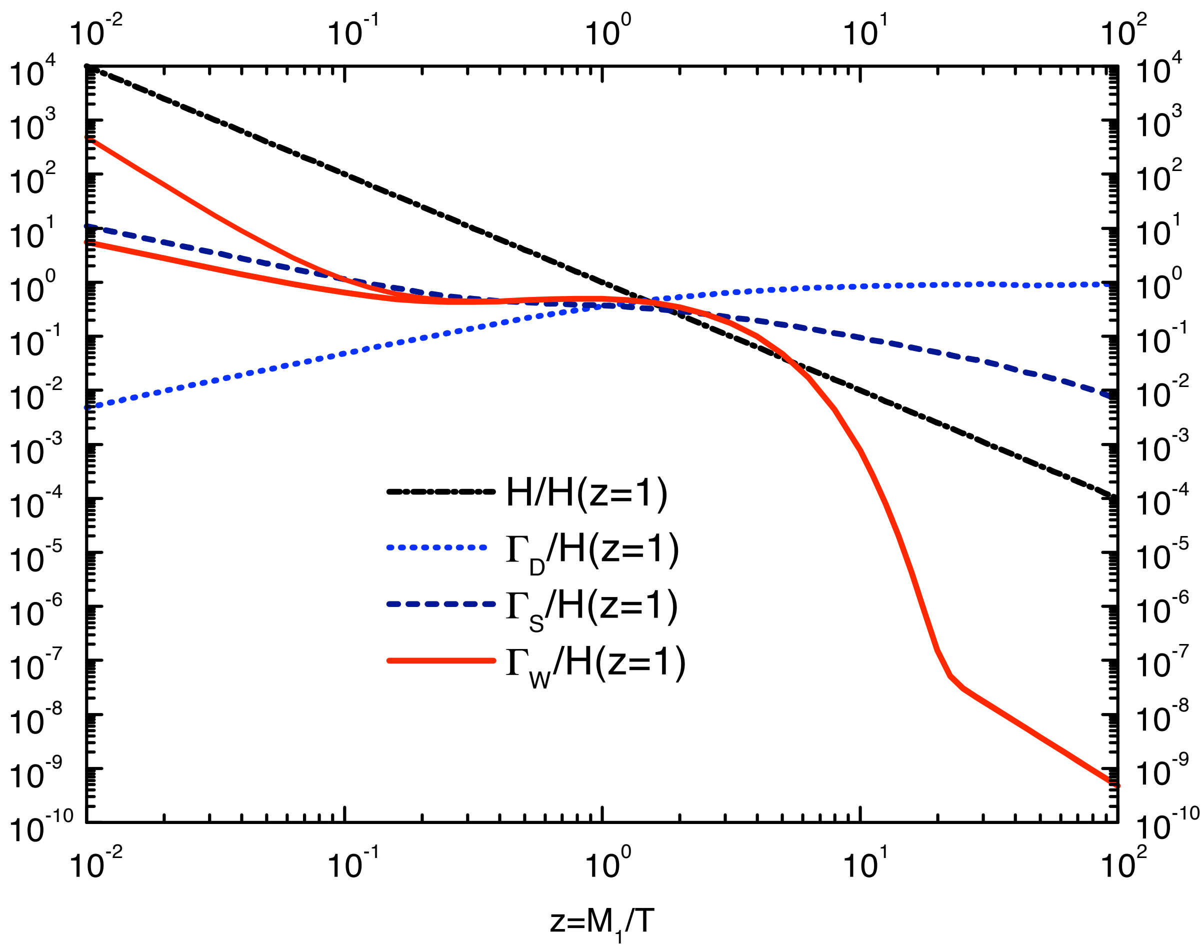
Example #1: typical set of parameters

$\varepsilon_1 = 10^{-6}$, $M_1 = 10^{10}$ GeV, $\tilde{m}_1 = 10^{-3}$ eV, $\bar{m} = 0.05$ eV

Interaction rates normalized to expansion rate:

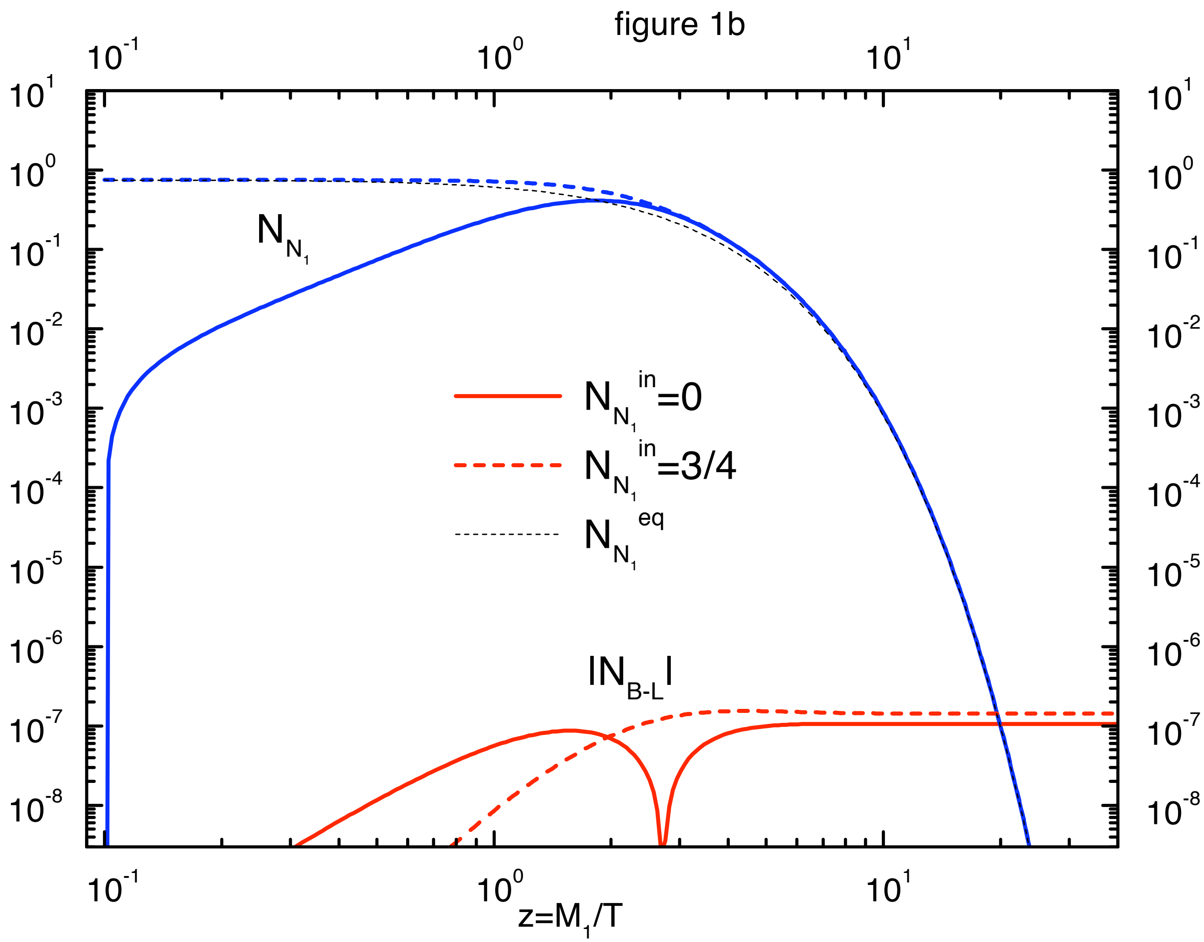
figure 1a



Example #1: typical set of parameters

$\varepsilon_1 = 10^{-6}$, $M_1 = 10^{10}$ GeV, $\tilde{m}_1 = 10^{-3}$ eV, $\bar{m} = 0.05$ eV

Evolution of N_1 abundance and $B-L$ asymmetry:

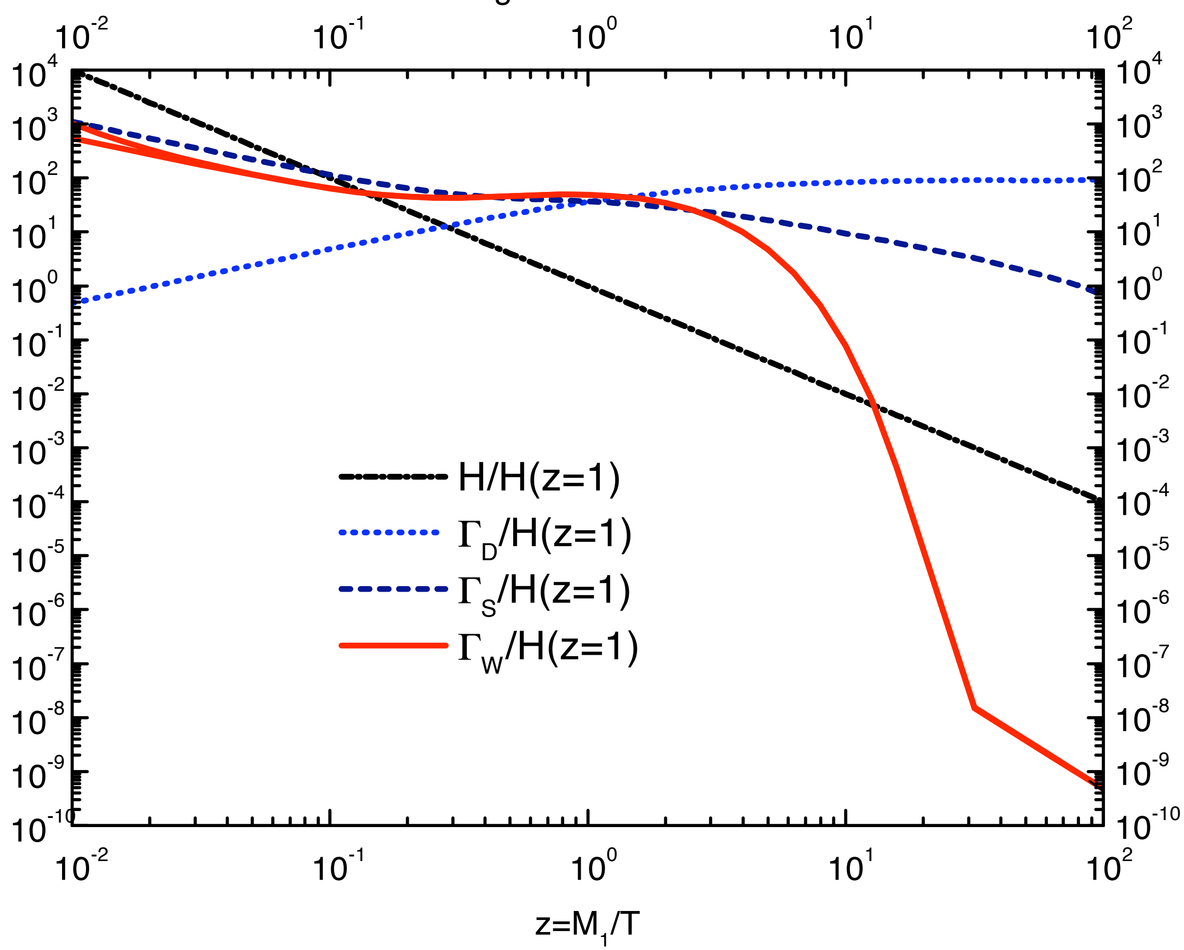


Example #2: larger $\tilde{m}_1 \rightarrow$ efficient N production, strong washout

$\varepsilon_1 = 10^{-6}$, $M_1 = 10^{10}$ GeV, $\tilde{m}_1 = 0.1$ eV, $\bar{m} = 0.05$ eV

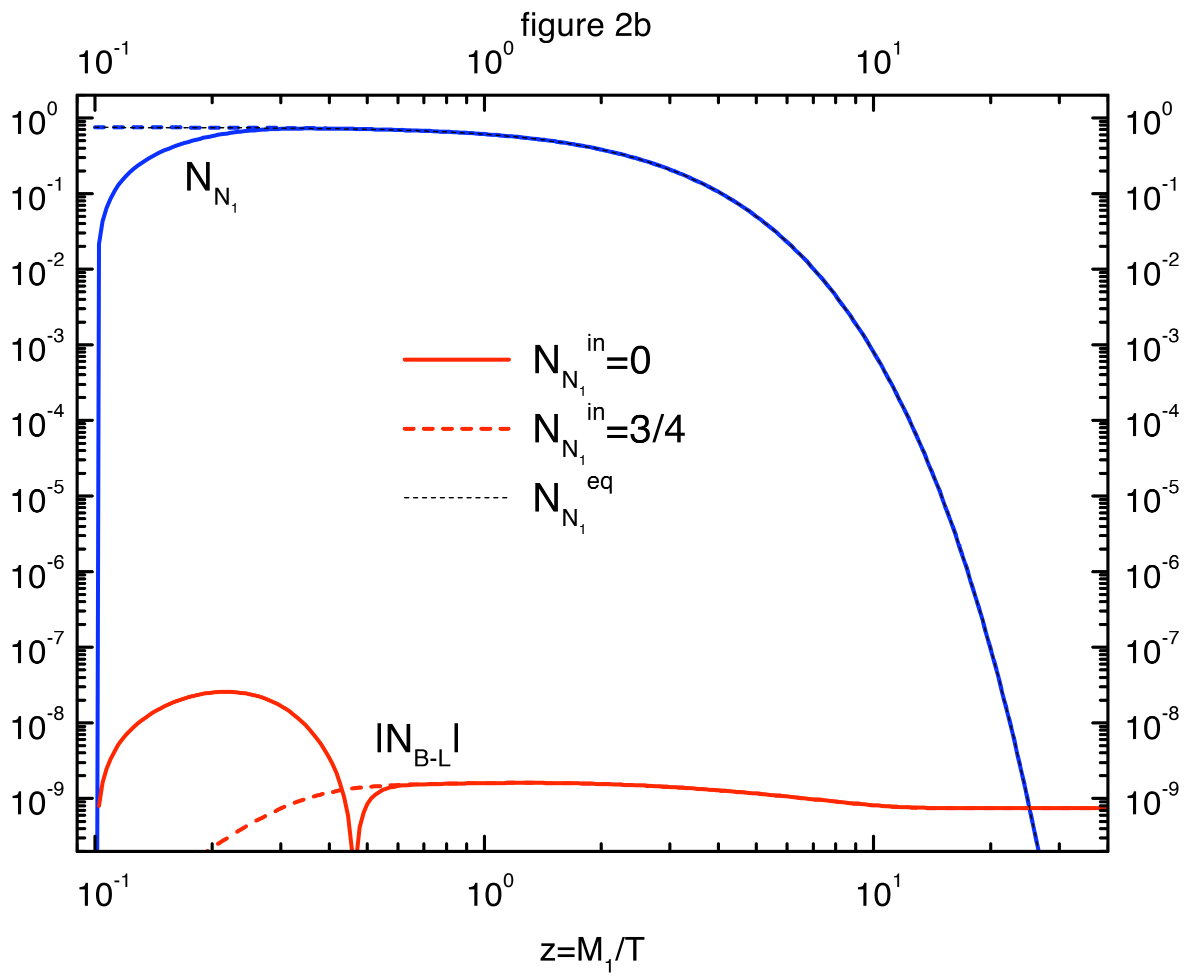
Interaction rates normalized to expansion rate:

figure 2a



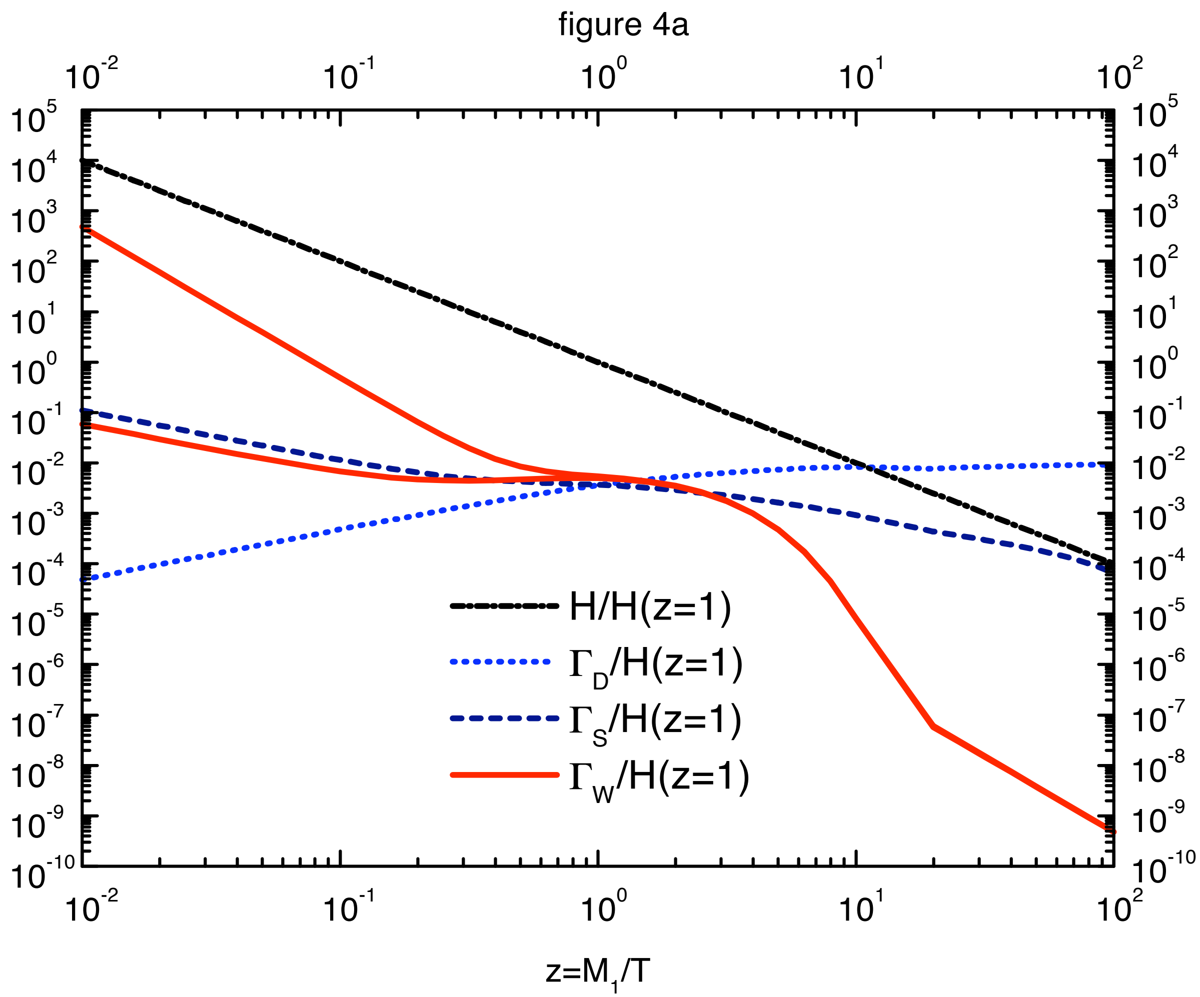
Example #2: larger $\tilde{m}_1 \rightarrow$ asymmetry reduced by ~ 100

$\varepsilon_1 = 10^{-6}$, $M_1 = 10^{10}$ GeV, $\tilde{m}_1 = 0.1$ eV, $\bar{m} = 0.05$ eV
Evolution of N_1 abundance and $B-L$ asymmetry:



Example #3: smaller $\tilde{m}_1 \rightarrow$ interactions too weak to bring system in thermal equilibrium

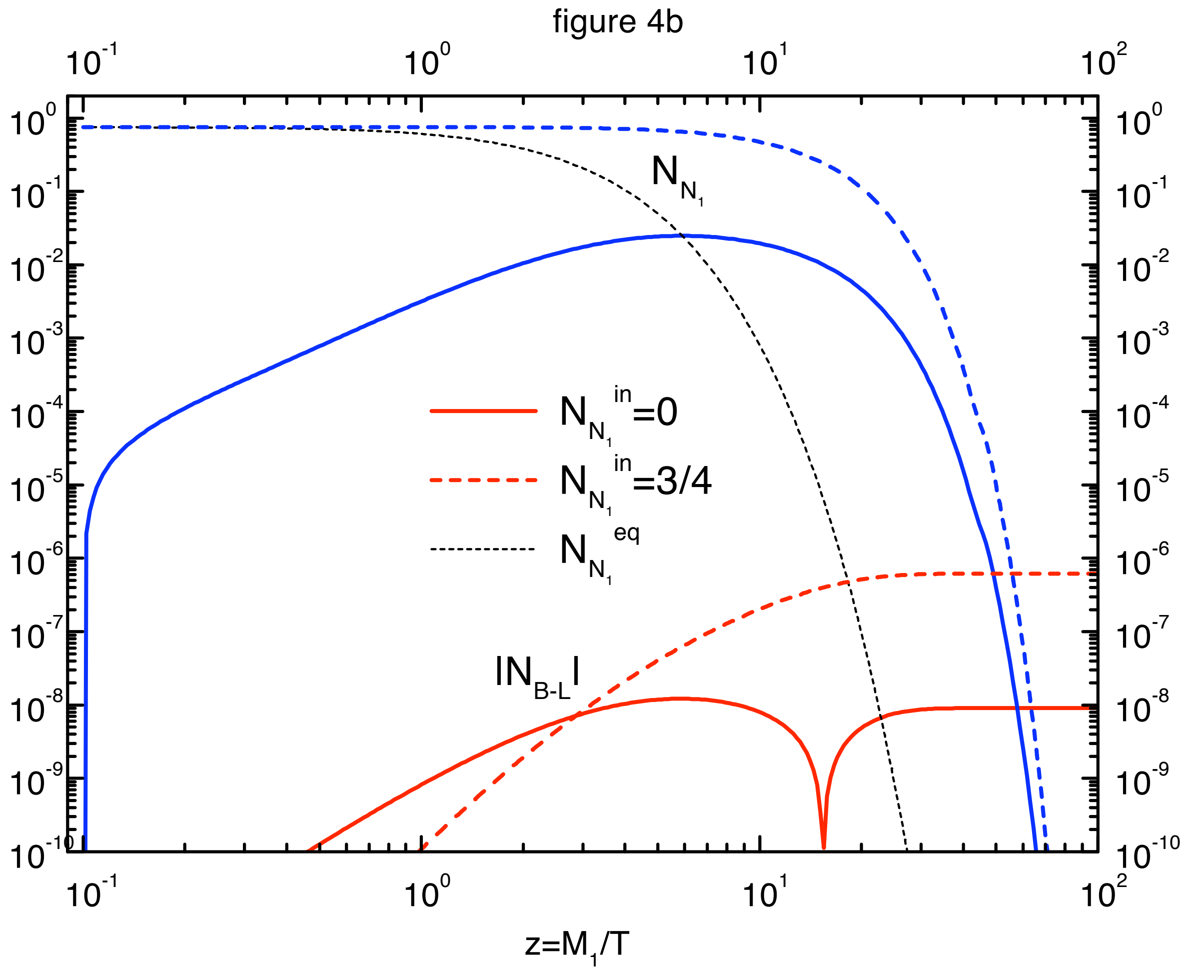
$\varepsilon_1 = 10^{-6}$, $M_1 = 10^{10}$ GeV, $\tilde{m}_1 = 10^{-5}$ eV, $\bar{m} = 0.05$ eV
Interaction rates normalized to expansion rate:



Example #3: smaller $\tilde{m}_1 \to N$ never in equilibrium, asymmetry reduced by ~ 10 compared to #1

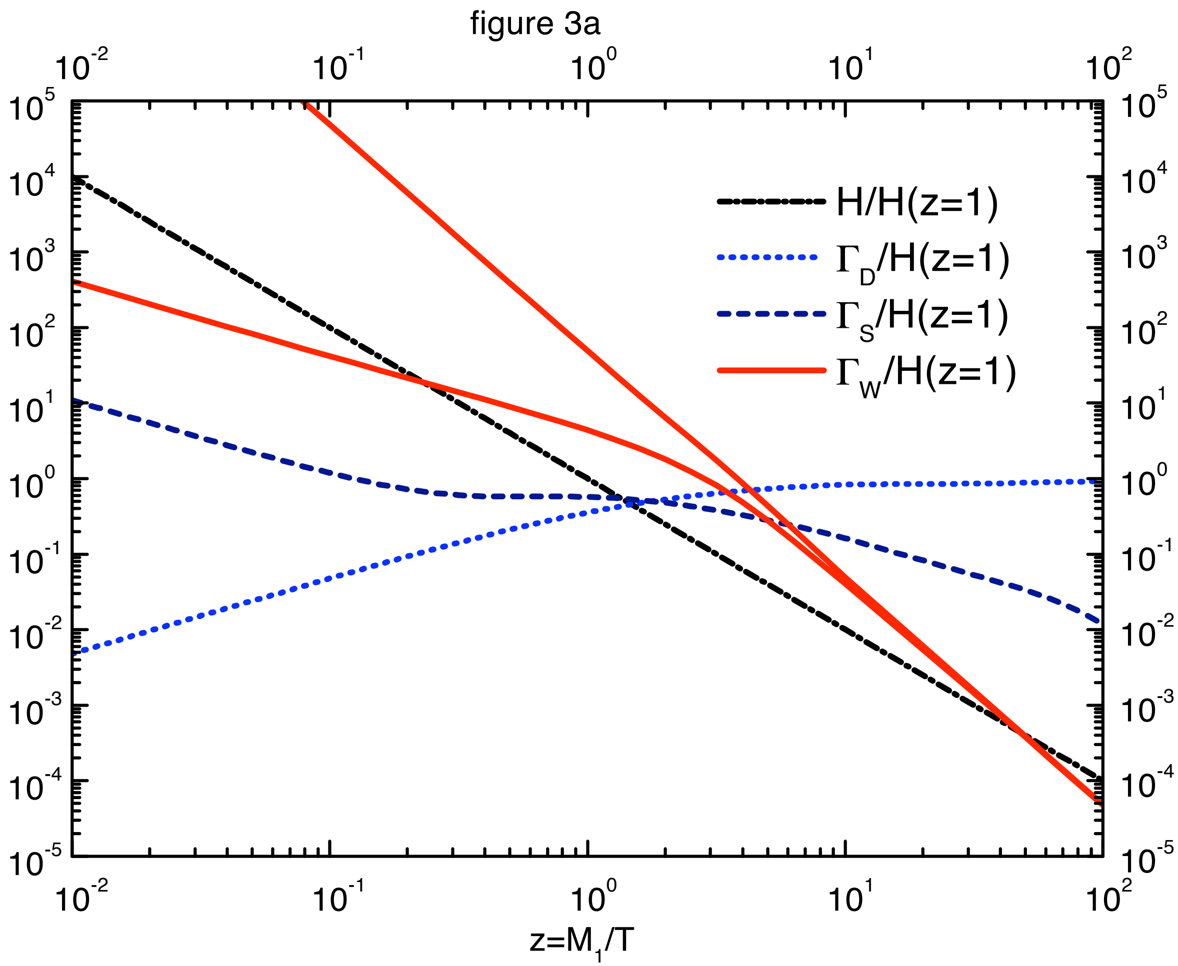
$\varepsilon_1 = 10^{-6}$, $M_1 = 10^{10}$ GeV, $\tilde{m}_1 = 10^{-5}$ eV, $\bar{m} = 0.05$ eV

Evolution of N_1 abundance and $B-L$ asymmetry:



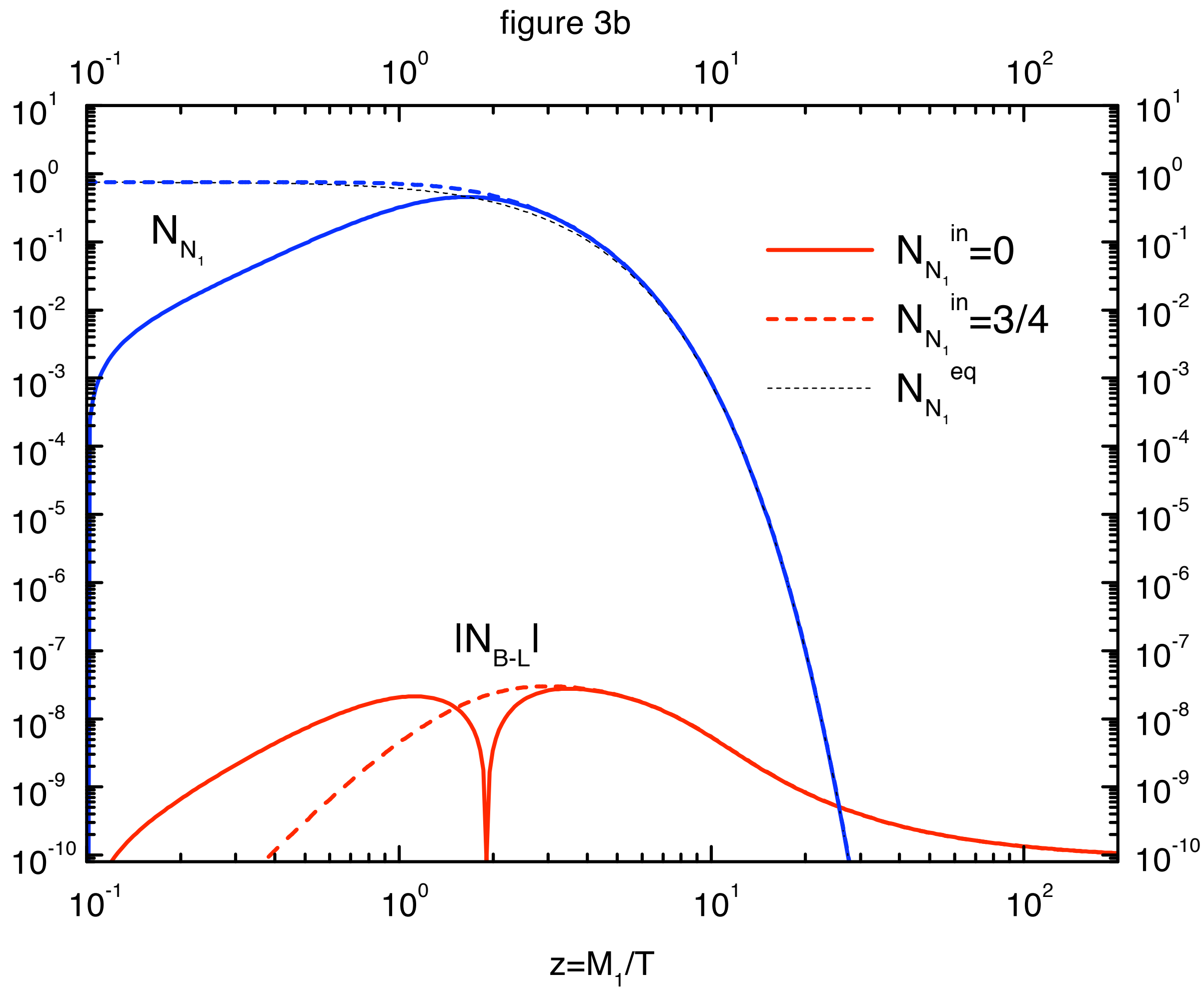
Example #4: larger M_1 $\rightarrow$ washout $\Gamma_{\Delta L=2} \propto M_1 \overline{m}^2$ increased by five orders of magnitude

$\varepsilon_1 = 10^{-6}$, $M_1 = 10^{15}$ GeV, $\tilde{m}_1 = 10^{-3}$ eV, $\bar{m} = 0.05$ eV
Interaction rates normalized to expansion rate:



Example #4: larger $M_1 \to$ asymmetry reduced by three orders of magnitude compared to #1

$\varepsilon_1 = 10^{-6}$, $M_1 = 10^{15}$ GeV, $\tilde{m}_1 = 10^{-3}$ eV, $\bar{m} = 0.05$ eV
Evolution of N_1 abundance and $B-L$ asymmetry:



Baryon asymmetry determined by four parameters

1. CP asymmetry ε_1
2. mass of decaying neutrino M_1
3. effective light neutrino mass $\widetilde{m}_1$ ($\propto$ decay width of N_1)
4. light neutrino masses $\overline{m} = \sqrt{m^2_{\nu_1} + m^2_{\nu_2} + m^2_{\nu_3}}$

Final baryon asymmetry

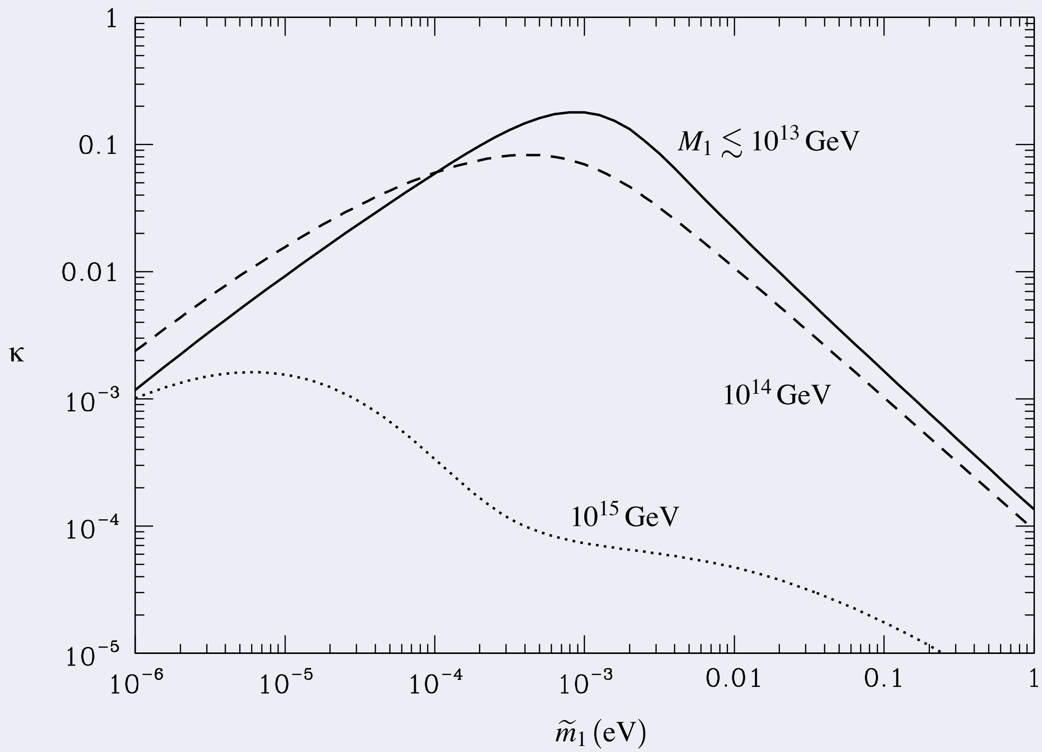
$$\eta_B \simeq 10^{-2}\, \varepsilon_1\, \kappa(\widetilde{m}_1, M_1 \overline{m}^2)$$

need to know:

- CP asymmetry ε_1 (from neutrino mass model)
- efficiency factor κ parametrizes N interactions (from integration of Boltzmann eqs.)
  (Barbieri et al. '00; Buchmüller, Di Bari & M.P. '02)

Efficiency factor κ as function of $\widetilde{m}_1$

(M.P. '96; Buchmüller, Di Bari & M.P. '02)



hierarchical light νs: $\overline{m} = 0.05$ eV

maximal efficiency:

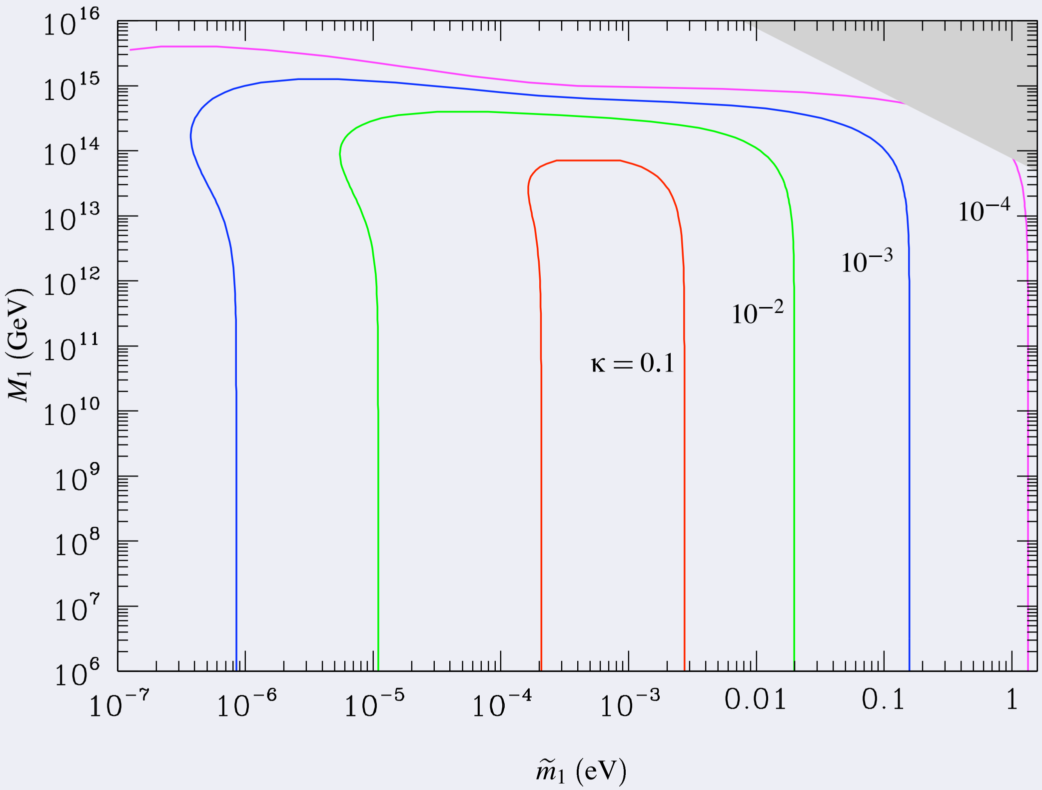
$\kappa^{\max} \simeq 0.18$

for $\widetilde{m}_1 \simeq 10^{-3}$ eV and $M_1 \lesssim 10^{13}$ GeV

$\longrightarrow$ N interactions reduce efficiency:

- for $\widetilde{m}_1 \ll 10^{-3}$ eV: N production inefficient
- for $\widetilde{m}_1 \gg 10^{-3}$ eV: washout too strong
- for $M_1 \gtrsim 10^{13}$ GeV: $\Gamma_{\Delta L=2} \propto M_1 \overline{m}^2$ becomes important

lines of constant κ in $(\widetilde{m}_1, M_1)$ plane



hierarchical light ν's:
$\overline{m} = 0.05\,\mathrm{eV}$

maximal efficiency in the mass range

$$10^{-4}\,\mathrm{eV} \lesssim \widetilde{m}_1 \lesssim 10^{-2}\,\mathrm{eV}$$

$$M_1 \lesssim 10^{13}\,\mathrm{GeV}$$

Baryon asymmetry determined by four parameters

1. CP asymmetry ε_1
2. mass of decaying neutrino M_1
3. effective light neutrino mass $\widetilde{m}_1$ ($\propto$ decay width of N_1)
4. light neutrino masses $\overline{m} = \sqrt{m_{\nu_1}^2 + m_{\nu_2}^2 + m_{\nu_3}^2}$

Final baryon asymmetry

$$\eta_B \simeq 10^{-2}\,\varepsilon_1\,\kappa(\widetilde{m}_1, M_1\overline{m}^2)$$

need to know:

- CP asymmetry ε_1 (from neutrino mass model)
- efficiency factor κ parametrizes N interactions (from integration of Boltzmann eqs.)
  (Barbieri et al. ’00; Buchmüller, Di Bari & M.P. ’02)

Leptogenesis vs. light neutrino parameters

Neutrino Yukawa Lagrangian:

$$\mathscr{L} = \bar{l}_L \, h \, \nu_R \, H^\dagger + \frac{1}{2} \bar{\nu}_R^c \, M \, \nu_R$$

Dirac neutrino masses: $m_D = h \, v$, $v = \langle H \rangle$

Without loss of generality: $M = \mathrm{diag}(M_1, M_2, M_3)$

light neutrino masses: $m_\nu = -m_D \, \frac{1}{M} \, m_D^T$

diagonalized by light neutrino mixing matrix U

$$U^\dagger \, m_\nu \, U^* = -D_m = -\mathrm{diag}(m_1, m_2, m_3)$$

Plug into seesaw formula

$$D_m = -U^\dagger\, m_\nu\, U^* = v^2\, U^\dagger\, h\, \frac{1}{M}\, h^T\, U^*$$

Multiply from the left and the right with $D_m^{-1/2}$

$$\begin{aligned} 1 &= v^2\, D_m^{-1/2}\, U^\dagger\, h\, \frac{1}{M}\, h^T\, U^*\, D_m^{-1/2} \\ &= v\, D_m^{-1/2}\, U^\dagger\, h\, M^{-1/2} \left(v\, D_m^{-1/2}\, U^\dagger\, h\, M^{-1/2} \right)^T \end{aligned}$$

We've constructed a complex orthogonal matrix Ω (Casas, Ibarra '01):

$$\Omega = v\, D_m^{-1/2}\, U^\dagger\, h\, M^{-1/2}\,, \qquad \Omega\Omega^T = 1$$

In the basis where both M and m_ν are diagonal:

$$\tilde{h} = U^\dagger\, h \quad \Rightarrow \quad \Omega = v\, D_m^{-1/2}\, \tilde{h}\, M^{-1/2}\,, \quad \Omega_{ij} = \frac{v}{\sqrt{m_i M_j}}\, \tilde{h}_{ij}$$

CP asymmetry

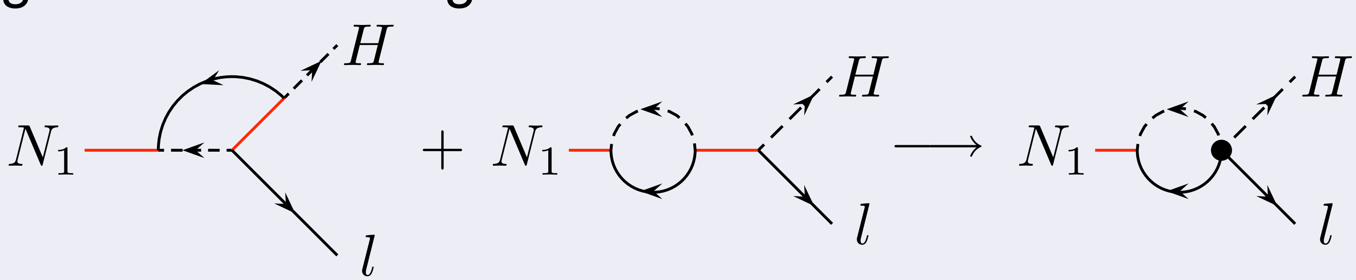
$$\varepsilon_1 = \frac{\Gamma(N\to l) - \Gamma(N\to \bar{l})}{\Gamma(N\to l) + \Gamma(N\to \bar{l})}$$

for $M_{2,3} \gg M_1$: upper bound on ε_1 in terms of light ν masses:

(Davidson & Ibarra '02; Buchmüller, Di Bari & M.P. '03; Hambye et al. '03)

$$\begin{aligned}\varepsilon_1 &\simeq -\frac{3}{16\pi}\frac{1}{(h^\dagger h)_{11}}\sum_i \mathrm{Im}\left[(h^\dagger h)^2_{1i}\right]\frac{M_1}{M_i} \\ &= \frac{3}{16\pi v^2}\frac{M_1}{(\tilde{h}^\dagger \tilde{h})_{11}}\sum_i m_i \mathrm{Im}\left(\tilde{h}_{i1}\right)^2\end{aligned}$$

It's obvious that ν masses enter. Integrate out heavy neutrinos in diagrams contributing to CP violation:



Making use of the orthogonality of Ω

Lower limit $\tilde{m}_1 \geq m_1$:

$$\tilde{m}_1 = \frac{v^2}{M_1}\left(\tilde{h}^\dagger \tilde{h}\right)_{11} = \sum_i m_i \left|\Omega_{i1}^2\right| \geq m_1 \sum_i \left|\Omega_{i1}^2\right| \geq m_1 \left|\sum_i \Omega_{i1}^2\right| = m_1$$

CP asymmetry in terms of light neutrino masses

$$\Omega^T \Omega = 1 \qquad \Rightarrow \qquad \mathrm{Im}\left(\Omega^T \Omega\right)_{11} = 0$$

From the definition of Ω it follows that

$$\sum_i \frac{1}{m_i} \mathrm{Im}\left(\tilde{h}_{i1}\right)^2 = 0$$

$$\Rightarrow \qquad \sum_i m_i \mathrm{Im}\left(\tilde{h}_{i1}\right)^2 = \sum_{i \neq 1} \frac{\Delta m_{i1}^2}{m_i} \mathrm{Im}\left(\tilde{h}_{i1}\right)^2$$

Maximal CP asymmetry

$$\varepsilon_1 = \frac{3}{16\pi} \frac{M_1}{v^2} \sum_{i \neq 1} \frac{\Delta m^2_{i1}}{m_i} \frac{\mathrm{Im}\left(\tilde{h}_{i1}\right)^2}{(\tilde{h}^\dagger \tilde{h})_{11}}$$

$\Rightarrow$ maximal CP asymmetry if $\mathrm{Im}\left(\tilde{h}_{31}\right)^2 / (\tilde{h}^\dagger \tilde{h})_{11}$ is maximal

$$\Rightarrow \ \varepsilon_1^{\max} = \frac{3}{16\pi} \frac{M_1 m_3}{v^2} \left[1 - \frac{m_1}{m_3}\sqrt{1 + \frac{\Delta m^2_{31}}{\tilde{m}^2_1}}\right]$$

two interesting limiting cases:

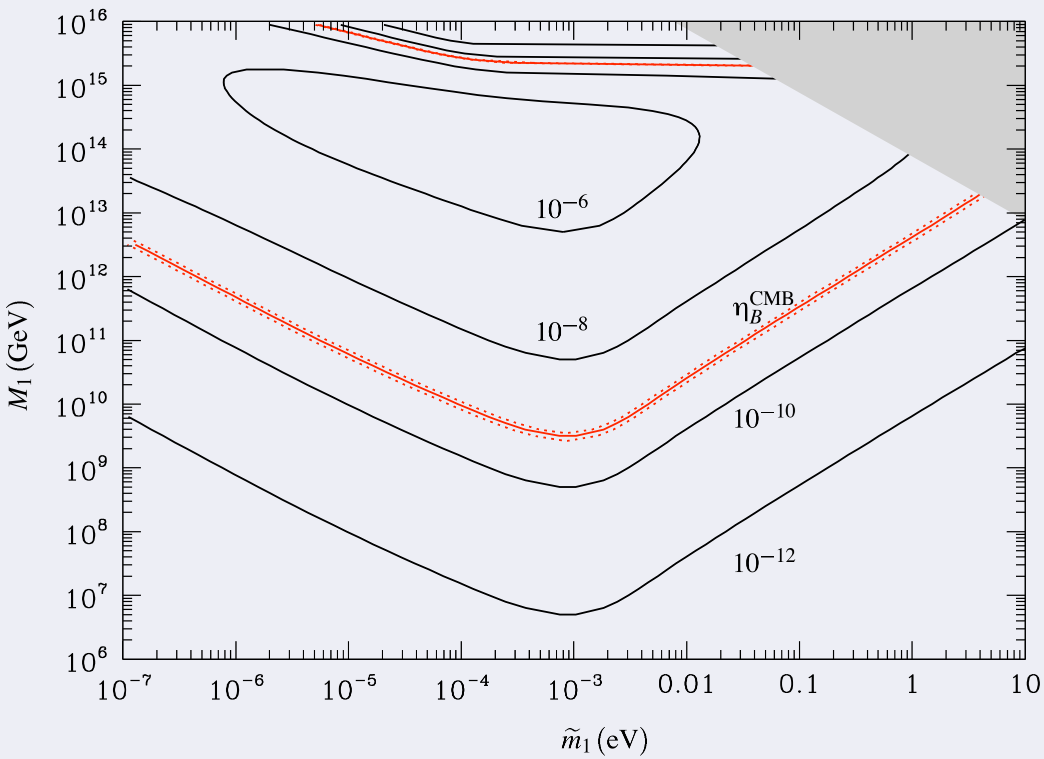
- hierarchical light νs: $m_1 \to 0 \quad \Rightarrow \quad \varepsilon_1^{\max} = \frac{3}{16\pi} \frac{M_1 m_3}{v^2}$
- degenerate light νs: $m_3^2 \gg \Delta m^2_{31} \quad \Rightarrow \quad \varepsilon_1^{\max} \to 0$

$\rightarrow$ *CP* asymm. suppressed if light ν spectrum quasi-degenerate

Maximal baryon asymmetry

$$\eta_B^{\max} = 10^{-2}\,\varepsilon_1^{\max}\,\kappa(\widetilde{m}_1, M_1\overline{m}^2)$$

hierarchical light νs: $\overline{m} = 0.05\,\text{eV} \quad \Rightarrow \quad \eta_B^{\max} = 10^{-2}\,\frac{3}{16\pi}\,\frac{M_1 m_{\nu_3}}{v^2}\,\kappa$



$\Rightarrow$ Lower bound on the baryogenesis temperature

$T_B \sim M_1 \gtrsim 10^9$ GeV

$t_B \sim 10^{-25}$ s

Constraints on neutrino parameters

1. N_1 production processes $\propto \widetilde{m}_1 \Rightarrow$ lower limit on $\widetilde{m}_1$
2. Washout processes:

   res. contrib. from $N_1 \propto \widetilde{m}_1 \Rightarrow$ upper limit on $\widetilde{m}_1$

   remainder $\propto M_1 \overline{m}^2 \Rightarrow$ upper limit on M_1 for fixed $\overline{m}$
3. maximal CP asymmetry $\propto M_1 \Rightarrow$ lower limit on M_1
   since $\eta_B \propto \varepsilon_1$

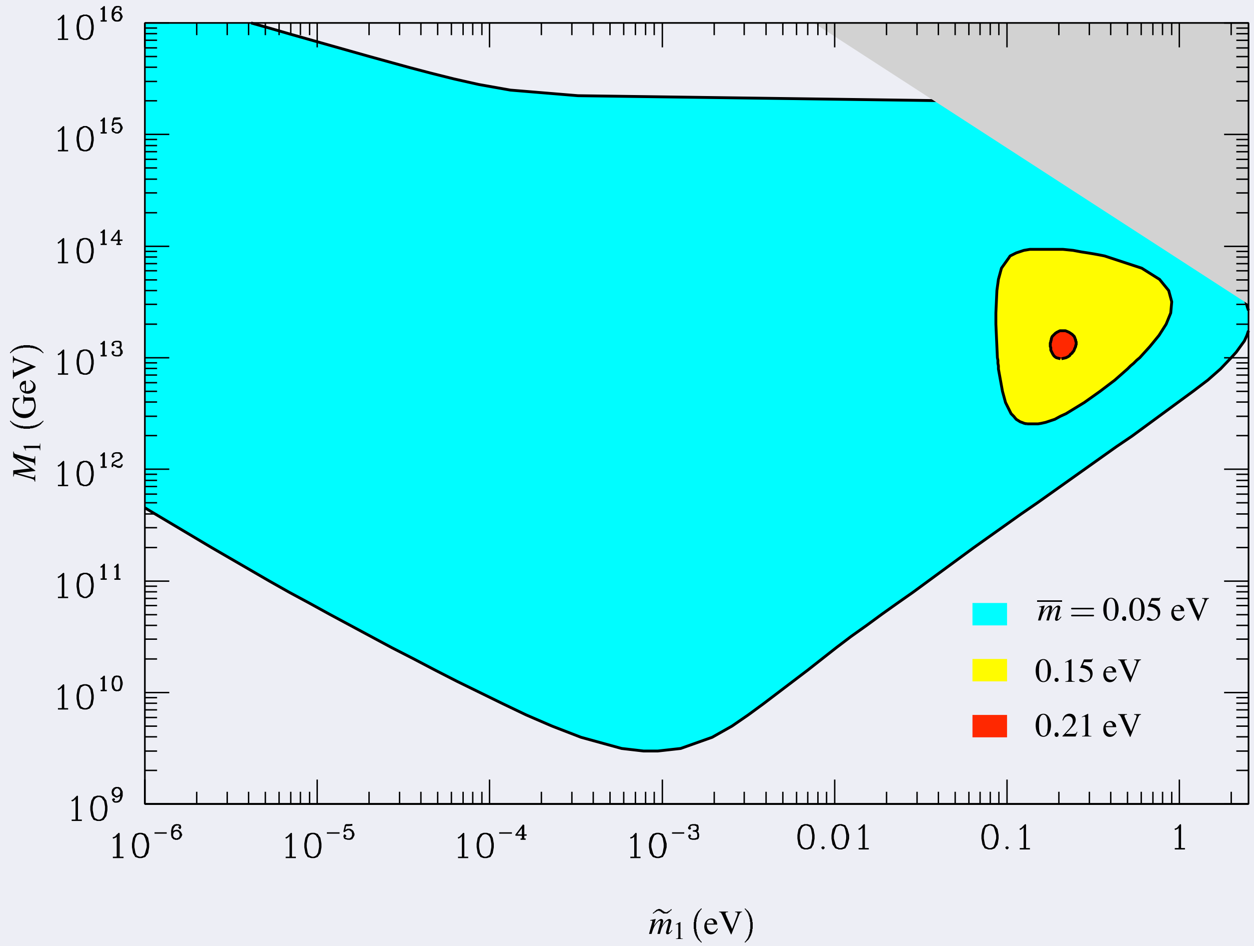
for fixed $\overline{m} \Rightarrow$ allowed region in $(\widetilde{m}_1, M_1)$ plane

Size of allowed region depends on $\overline{m}$ since:

- max. CP asymm. suppressed for quasi-degenerate light νs
- $\widetilde{m}_1 \geq m_{\nu_1}$

$\Rightarrow$ upper bound on $\overline{m}$

(Buchmüller, Di Bari & M.P. '03, '04)



light ν masses: $\overline{m} < 0.22\,\mathrm{eV} \quad \Rightarrow \quad m_{\nu_i} < 0.13\,\mathrm{eV}$

RHN masses: $T_B \sim M_1 \gtrsim 10^9$ GeV

The neutrino mass window for baryogenesis

- upper bound on light ν masses $m_{\nu_i} \lesssim 0.1\,\mathrm{eV}$
- no dependence on initial conditions for $\widetilde{m}_1 \gtrsim 10^{-3}\,\mathrm{eV}$

since $\widetilde{m}_1 \geq m_{\nu_1} \to$ leptogenesis window for neutrino masses

$$10^{-3}\,\mathrm{eV} \lesssim m_{\nu_i} \lesssim 0.1\,\mathrm{eV}$$

compatible with ν oscillations $(m_{\mathrm{atm}} \sim 0.05\,\mathrm{eV})$

Analytical solution for efficiency factor in leptogenesis window:

$$\kappa = (2 \pm 1) \times 10^{-2} \left(\frac{0.01\,\mathrm{eV}}{\widetilde{m}_1} \right)^{1.1 \pm 0.1}$$

What have we learned?

- Type I seesaw naturally explains the cosmological baryon asymmetry and the smallness of neutrino masses
- Quasi-degenerate light ν masses are incompatible with leptogenesis:

$$m_{\nu_i} < 0.13\ \mathrm{eV}$$

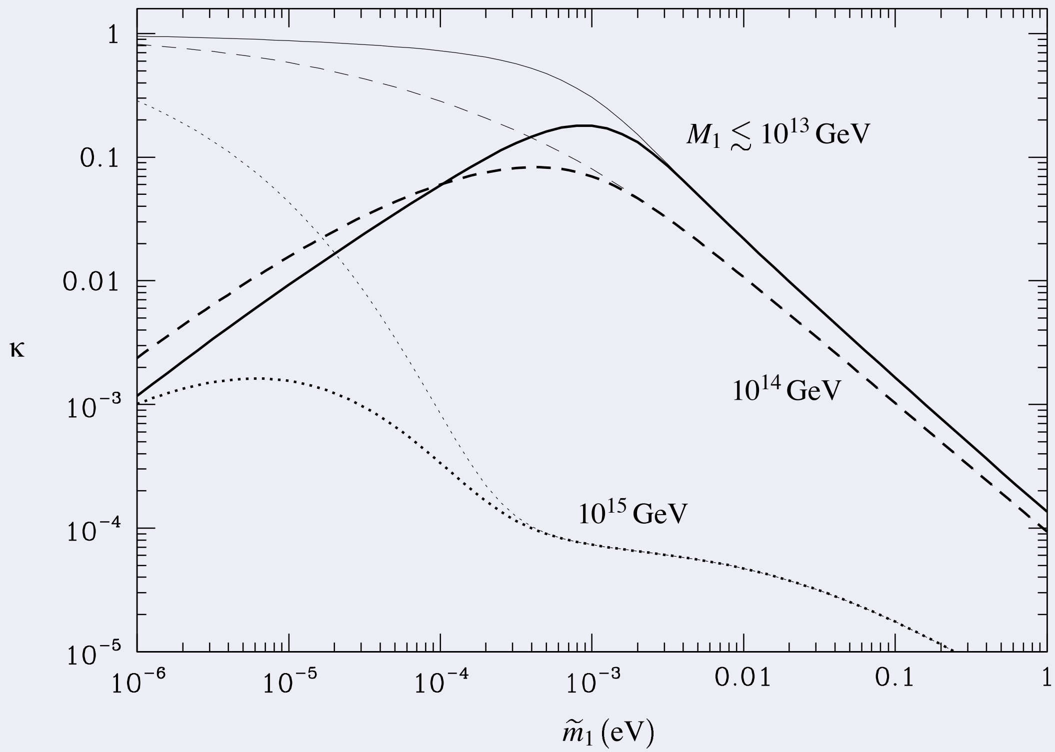
- lower bound on the baryogenesis temperature:

$$T_B \gtrsim 10^9\ \mathrm{GeV}\ , \qquad t_B \sim 10^{-25}\ \mathrm{s}$$

How reliable are these bounds?

Can they be evaded?

Initial conditions: Neutrino production?



hierarchical light νs:
$\overline{m} = 0.05$ eV

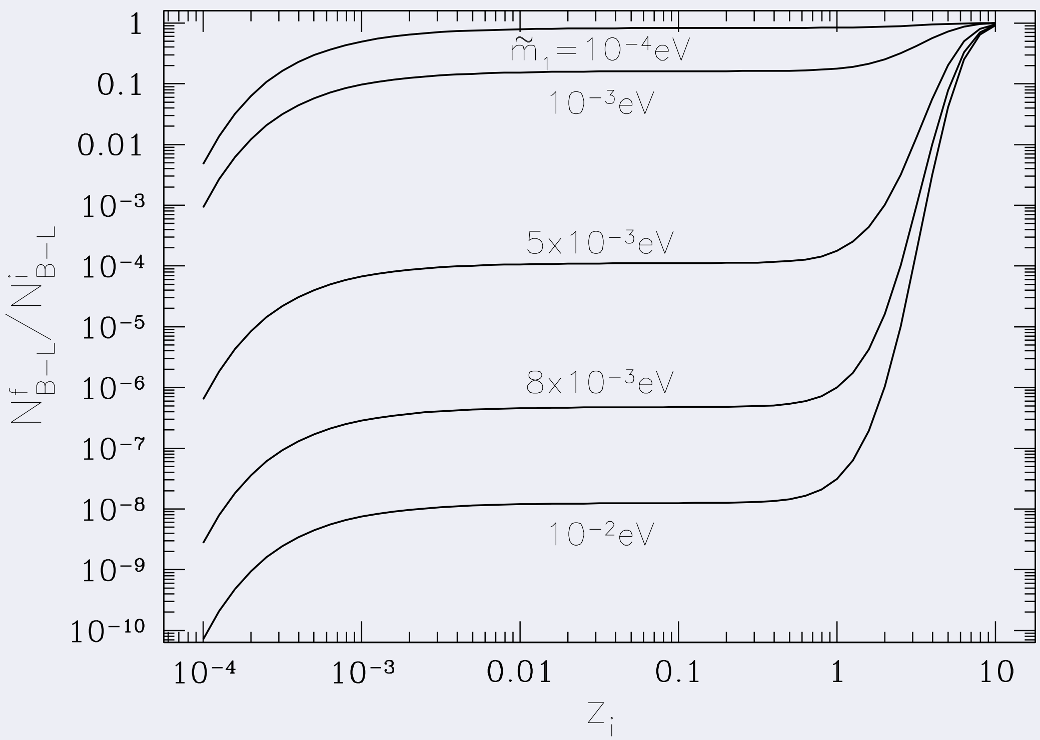
initial conditions

- $N_{N_1} = N_{N_1}^{\text{eq}}$ at $T \gg M_1$: thin lines
- $N_{N_1} = 0$ at $T \gg M_1$: thick lines

no dependence on initial conditions for $\widetilde{m}_1 \gtrsim 10^{-3}$ eV

Initial conditions: Primordial Asymmetry?

initial asymmetry before leptogenesis:
effect of washout?



Washout factor for hierarchical light νs:
$\overline{m} = 0.05\,$eV
and
$M_1 = 10^{10}\,$GeV

Initial temperature:
$z_i = \dfrac{M_1}{T_i}$

efficient washout of initial asymmetry at $z_i \sim 1$ for $\widetilde{m}_1 \gtrsim 10^{-3}\,$eV

no dependence on initial conditions for $\widetilde{m}_1 \gtrsim 5 \times 10^{-3}\,$eV

Non-thermal leptogenesis (with F. Hahn-Woernle)

Up to now we've assumed that everything happens in the thermal, radiation dominated phase of the early universe.

Alternative: non-thermal leptogenesis at the end of inflation

Simple model: inflaton Φ decays entirely into RHNs N_1

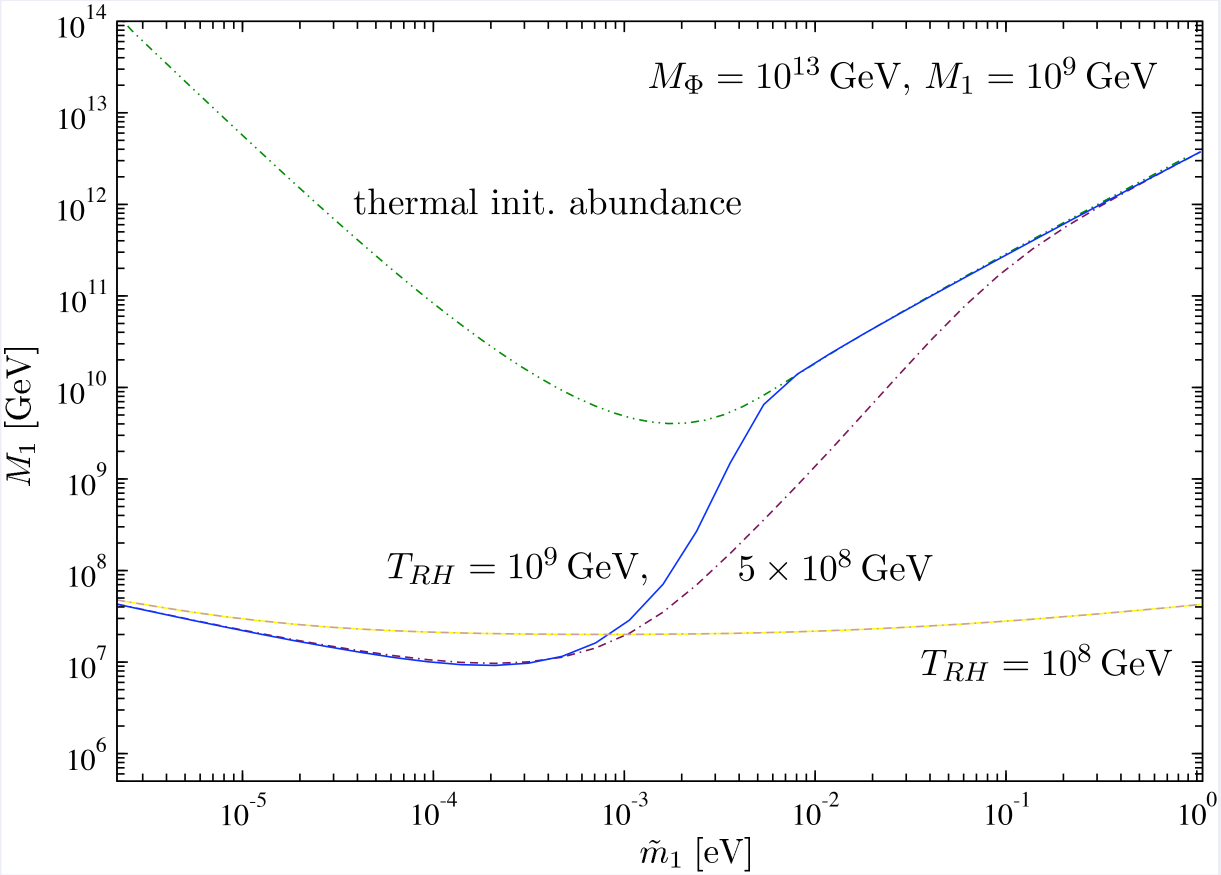
Reheating temperature $T_{RH} \propto \sqrt{\Gamma_\Phi M_{Pl}}$ is now a measure for the inflaton-neutrino coupling and NOT a physical temperature.

The actual reheating takes place when N_1 decays into SM particles which rapidly thermalize
$\rightarrow$ physical reheating temperature: $T_{RH}^N \propto \sqrt{\Gamma_N M_{Pl}}$ can be much smaller than M_N, i.e. that's a way to reduce washout without getting into trouble with N-production

Drawback: much more model dependent

Lower bound on M_1 in this scenario

$T_{RH} \propto$ inflaton-neutrino coupling,
physical reheating temperature $\propto \sqrt{\tilde{m}_1 M_{Pl}}$



Lower bound on M_1 relaxed by several orders of magnitude!

Kinetic vs. thermal equilibrium

Lower limit on M_1 from weak washout regime $(\widetilde{m}_1 \ll 10^{-3}\,\mathrm{eV})$

$\rightarrow$ strong dependence on initial conditions and on how system approaches thermal equilibrium

Usual calculations rely on approximations:

1. Maxwell-Boltzmann statistics
2. Kinetic equilibrium: $f(E) = \frac{n}{n^{\mathrm{eq}}} \mathrm{e}^{-E/T}$

Scattering cross sections are energy-dependent, i.e. assuming kinetic equilibrium seems questionable.

Need to study how the system approaches equilibrium and how that depends on different assumptions (with F. Hahn-Wörnle).

Thermal corrections

Leptogenesis takes place in a thermal bath

$\rightarrow$Thermal corrections have to be considered, should regulate IR divergences

Controversial results, based on high temperature approximations, in literature (Covi et al., ’98; Giudice et al., ’03)

Need to compute scattering and decay rates in finite temperature field theory (with A. Brandenburg and F. Steffen)

Problem: two limiting cases considered in literature

1. thermal corrections for heavy states, i.e. $T \ll M$
2. thermal effects for massless fields, i.e. $T \gg M$

Relevant regime for leptogenesis: $T \sim M$

Alternatives?

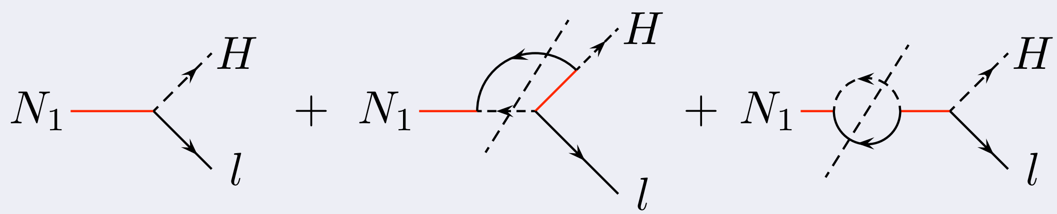
What if light neutrinos are quasi-degenerate?

What if the reheating temperature is lower than $\sim 10^9$ GeV?

- decouple light neutrino masses from baryogenesis, i.e. contribution to light ν masses and/or baryogenesis from triplet Higgs
  some other mechanism for light ν masses,...
- resonant leptogenesis, soft leptogenesis in SUSY models
- non-thermal leptogenesis, i.e. through inflaton decay or Affleck-Dine, ...

Resonant Leptogenesis

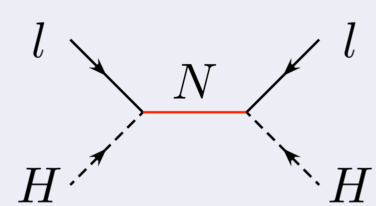
Resonant enhancement of CP-asymmetry for $M_{2,3} - M_1 \ll M_1$:



Almost no effect on bound on light ν masses, but lower limit on T_B, M_1 can be relaxed.
However: many different results in literature !?

Problem: N_i unstable, i.e. cannot appear as in- or out-states of S-matrix elements

Solution: scattering amplitudes of stable particles with N_i as intermediate states



Factorisation: effective one-loop couplings of N_i

Resummation of self-energies

regularizes resonant propagator $\Rightarrow$ mixing effects

$$\left(S^{-1}\right)_{ij} = \not{p} - M_i - \Sigma_{ij}$$

Renormalization known (Kniehl & Pilaftsis '96)

Chiral decomposition of propagator:

$$S = P_R S^{RR} + P_L S^{LL} + P_L \not{p} S^{LR} + P_R \not{p} S^{RL}$$

Contribute to different scattering processes:

$$\mathscr{M}(l_r \to \bar{l}_s) \propto h_{ri} S^{LL}_{ij} h_{sj} \qquad \mathscr{M}(\bar{l}_r \to l_s) \propto h^*_{ri} S^{RR}_{ij} h^*_{sj}$$

$$\mathscr{M}(l_r \to l_s) \propto h^*_{ri} S^{RL}_{ij} h_{sj} \qquad \mathscr{M}(\bar{l}_r \to \bar{l}_s) \propto h_{ri} S^{LR}_{ij} h^*_{sj}$$

Contributions of different N_i mass eigenstates?

Factorization (Anisimov, Broncano & M.P. '05):

Different methods:

1. Decompose scattering ampl. into partial fractions, e.g.:

$$\mathscr{M}(l_r \to \bar{l}_s) \propto \lambda_{r1} \frac{1}{p^2 - \hat{M}_1^2} \lambda_{s1} + \lambda_{r2} \frac{1}{p^2 - \hat{M}_2^2} \lambda_{s2} + \dots$$

   λ_{ri}: resummed effective N_i Yukawa coupling

   Consistency: all 4 amplitudes can be factorized simultaneously.

2. Diagonalization of propagators, e.g.: $U\, S^{LL}\, U^T = S^{\rm diag}$

$$\mathscr{M}(l_r \to \bar{l}_s) \propto \left(hU^T\right)_{ri} S^{\rm diag}_{ii} \left(hU^T\right)_{si}$$

   $\left(hU^T\right)_{ri}$: resummed effective N_i Yukawa coupling

   Consistency: for $p^2 = M_i^2$ all 4 amplitudes can be factorized simultaneously.

Results:

Both methods yield identical results for physical quantities:

1. Decay widths: $\Gamma(N_i \to \bar{l}_r) \propto |\lambda_{ri}|^2 = \left|\left(hU^T\right)_{ri}\right|^2$, for $p^2 = M_i^2$
2. CP-asymmetries, e.g.:

$$\varepsilon_1 \propto \frac{M_2^2 - M_1^2}{\left(M_2^2 - M_1^2\right)^2 + (M_2\Gamma_2 - M_1\Gamma_1)^2} ,$$

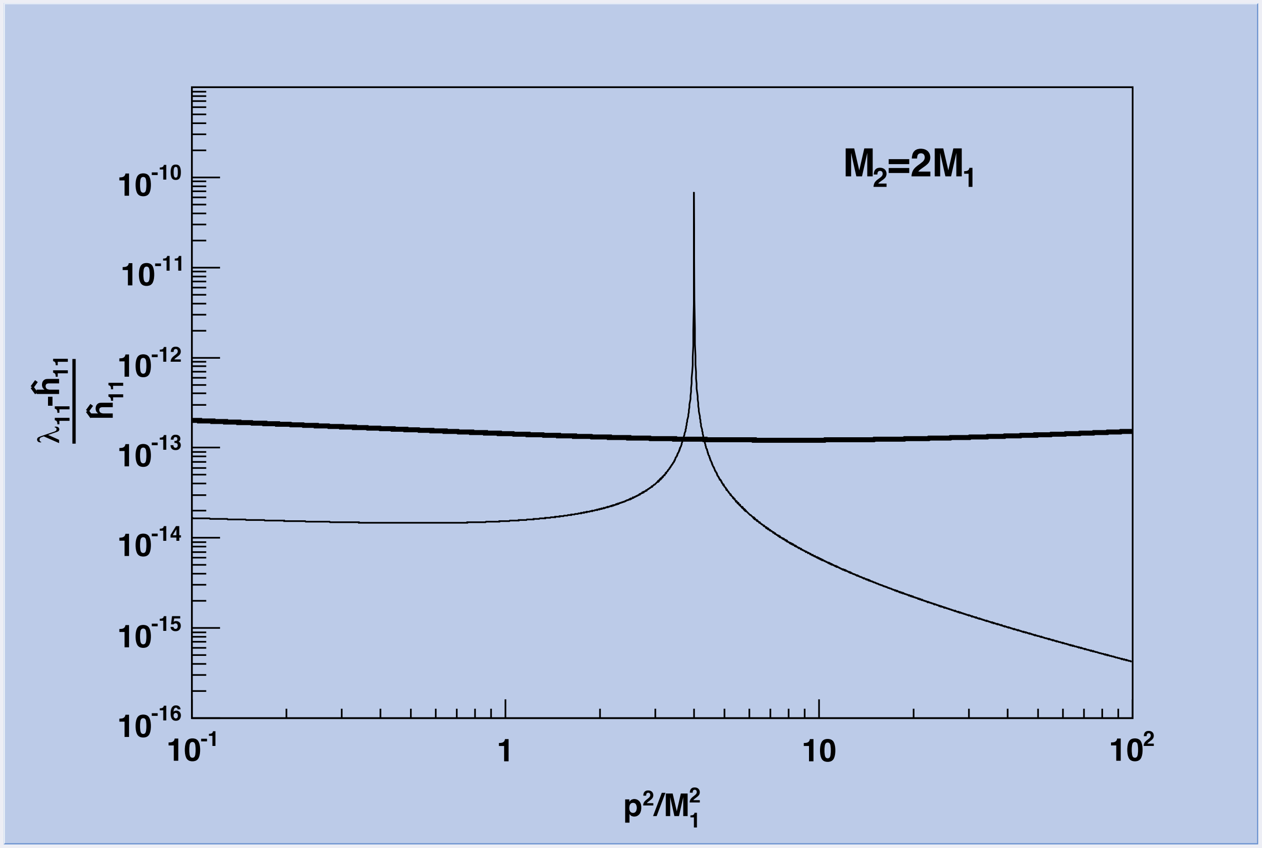
Previous approaches, e.g., resum only self-energy Σ_{jj} of intermediate neutrino $N_j \Rightarrow$ regulator: Γ_j (Pilaftsis & Underwood '04)

$$\varepsilon_1 \propto \frac{M_2^2 - M_1^2}{\left(M_2^2 - M_1^2\right)^2 + M_1^2\Gamma_2^2}$$

Different neutrino flavours are treated differently!

Relative one-loop correction to couplings of N_1

Our result (thick line) compared to the one of Pilaftsis et al.:



thin line has resonance at $p^2 = M_2^2$, i.e. contributions from different neutrino mass eigenstates not properly separated in previous approaches.

Conclusions

- Cosmology is not only a lot of fun but can also tell us a lot about particle physics
$\longrightarrow$ *cosmology is the continuation of particle physics by other means* (Carl von Clausewitz)
- Leptogenesis relates η_B to light neutrino properties if

$$m_{\nu_i} \lesssim 0.1 \text{ eV} \qquad \text{and} \qquad T_B \gtrsim 10^9 \text{ GeV}$$

- Leptogenesis works best in neutrino mass window

$$10^{-3}\,\text{eV} \lesssim m_{\nu_i} \lesssim 0.1\,\text{eV}$$

consistent with neutrino oscillations


MATTER
ANTI-MATTER
S. Harris